UNIVERSITÀ DEGLI STUDI FIRENZE

DISEI
DIPARTIMENTO DI SCIENZE PER L'ECONOMIA E L'IMPRESA

Working Papers - Economics

Decomposing the Welfare Gains from Trade

Paolo Bertoletti and Federico Etro

Working Paper N. 22/2024

DISEI, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze (Italia) www.disei.unifi.it

Decomposing the Welfare Gains from Trade

Paolo Bertoletti and Federico Etro[1]

University of Milan-Bicocca and University of Florence

October 2024

Keywords: Gains from trade, Monopolistic competition, Heterogeneous firms

JEL Codes: D11, D43, F12

Abstract

The textbook of international economics by Krugman, Obstfeld and Melitz (2022) introduces the gains from trade under monopolistic competition with a simple linear demand model. We use its representative consumer foundation to decompose the welfare gains from a market expansion into competitive effects, variety effects and selection effects.

[1] *Correspondence.* Paolo Bertoletti: Dept. of Economics, Management and Statistics, University of Milan-Bicocca, Piazza dell'Ateneo Nuovo 1, 20126 Milan, Italy. Phone: +39-02-64483105, email: *paolo.bertoletti@unimib.it.* Federico Etro: Florence School of Economics and Management, Via delle Pandette 32, Florence, 50127. Phone: +39-055-2759603, email: *federico.etro@unifi.it.* We gratefully acknowledge the insightful comments of an anonymous referee.

Krugman *et al.* (2022: Ch. 8) analyze a market of monopolistic competition by adopting the following linear demand for a firm with price p:

$$Q = S\left[\frac{1}{n} - b(p - p^*)\right], \tag{1}$$

where n is the number of competitors, p^* the average price, $b > 0$ an inverse measure of product differentiation and the constant S captures the market size. Notice that the choke price is $\frac{1}{nb} + p^*$ and the inverse demand has slope $1/bS$. Each firm produces a differentiated good with a marginal cost c and sets its price to maximize gross profits $\pi = (p - c)Q$ after paying an entry cost F.

This simple model is broadly inspired by the one of Salop (1979), which is a discrete choice model of Bertrand competition where consumers are heterogeneous in preferences and buy one out of n spatially differentiated goods.[2] It is also closely related to the monopolistic competition model of Melitz and Ottaviano (2008), which however adopts a different linear demand system, whose total demand decreases in the average price. In this note we complement Krugman *et al.* (2022: Ch. 8) by providing a representative consumer foundation for the demand system (1) that reflects a standard form of love for variety. We use it to decompose the welfare impact of an expansion of market size, which captures the opening up to costless trade, into competitive effects (lower prices), variety effects (more goods) and, under heterogeneous firms, selection effects (more efficient producers). We show that the competitive effects dominate the others, and the selection effects can be stronger than the variety effects.

[2] In particular, products are equidistantly located on a unit circle where the preferences of S consumers are uniformly distributed, and the demand of each product depends on the average price p^* of the two neighboring products.

1 Monopolistic competition and trade

Consider S consumers who share quasi-linear preferences represented by the following indirect utility over differentiated goods $j = 1, 2, ..., n$ with prices p_j:

$$V = \sum_{j=1}^{n} \frac{(A - bp_j)^2}{2b} - \frac{A}{b} + E, \tag{2}$$

where E is individual expenditure and A is a price aggregator defined as:

$$A \equiv \frac{1}{n} + bp^* \quad \text{with } p^* = \frac{1}{n}\sum_{j=1}^{n} p_j.$$

Since the impact of the aggregator on utility is null ($\partial V/\partial A = 0$), the individual demand for good i can be easily obtained by Roy's identity as $q_i = \left|\frac{\partial V}{\partial p_i}\right| = A - bp_i$, which provides the aggregate demand (1).[3] This entails a constraint on total individual demand, namely $\sum_{j=1}^{n} q_j = 1$: each consumer purchases a total quantity of differentiated goods that is constant. Also notice that A/b is equal to the choke price.

Under monopolistic competition when firms have a common marginal cost c, a firm i sets its price p_i to maximize:

$$\pi_i = (p_i - c)(A - bp_i)S,$$

taking as given the aggregator A (or equivalently the average price p^*). By demand linearity each price is then set as $p_i = \frac{1}{2}(c + \frac{A}{b})$. Using the definition of the aggregator and symmetry across firms, we obtain the common equilibrium price $p = c + \frac{1}{nb}$ with associated gross profits $\pi = \frac{S}{n^2 b}$. Then, free entry (namely, the zero-profit condition $\pi = F$) delivers the following number of firms and price:

$$n = \sqrt{\frac{S}{bF}} \quad \text{and} \quad p = c + \sqrt{\frac{F}{bS}},$$

[3] More general demand systems that depend on a common aggregator are used in Bertoletti and Etro (2024) to study the welfare properties of monopolistic competition in a quasi-linear environment.

which depend on market size according to:

$$\frac{dn}{dS} = \frac{1}{2}\sqrt{\frac{1}{bFS}} > 0 \quad \text{and} \quad \frac{dp}{dS} = -\frac{1}{2}\sqrt{\frac{F}{bS^3}} < 0. \tag{3}$$

As well known, in such a model an expansion of the market size attracts more firms and reduces prices (by decreasing the equilibrium value of the choke price).

From (2), the equilibrium value of utility can then be written as:

$$V = n\frac{(A-bp)^2}{2b} - \frac{A}{b} + E, \tag{4}$$

which represents the relevant welfare measure, since the equilibrium profits are null. A larger market size affects it only through changes of n and p, because the impact through A is null. Total differentiation of (4) provides the welfare impact of a market size expansion, $\frac{dV}{dS} = \frac{3}{4}\sqrt{\frac{F}{bS^3}}$, which is positive and decreasing with respect to S. We can decompose the welfare contribution of more competition and business creation as:[4]

$$\begin{aligned} \frac{dV}{dS} &= \underbrace{\frac{\partial V}{\partial p}\frac{dp}{dS}}_{\textit{competition}} + \underbrace{\frac{\partial V}{\partial n}\frac{dn}{dS}}_{\textit{variety}} \\ &= \frac{1}{2}\sqrt{\frac{F}{bS^3}} + \frac{1}{4}\sqrt{\frac{F}{bS^3}}. \end{aligned}$$

Accordingly, in larger markets consumers gain both from a price reduction and from an increase in the number of goods. However, the competitive effect of costless trade is here stronger and actually twice as important as the variety effect.

[4] The love for variety of our representative consumer implies stronger gains from variety than in the original Salop (1979) model. An implication is that the optimal number of firms $n^* = \sqrt{S/(2bF)}$ is larger than in the corresponding spatial model. Nevertheless, equilibrium entry remains excessive when firms have a common marginal cost.

2 Performance differences across producers

As suggested by Krugman *et al.* (2022: Ch. 8), the monopolistic competition model of the previous section can be extended to the case of heterogeneous firms, as in Melitz (2003) and Melitz and Ottaviano (2008). Let us assume that firms decide to enter and pay an entry cost $F > 0$ before knowing their productivity, on the basis of the expected profitability. Upon entry, each firm draws its marginal cost c from a continuous distribution $G(c)$ with density $g(c)$ on the support $[0, \bar{c}]$, and decides whether to actually produce and sell its variety at a price $p(c)$, or to remain inactive. Clearly, the mass n of entering firms that decide to become active corresponds to that of firms with a marginal cost below the choke price. Let us indicate with $\hat{c}$ this cutoff, assumed smaller than $\bar{c}$.

In this case the indirect utility function corresponding to (2) can be written as:

$$V = n \int_0^{\hat{c}} \frac{(A - bp(c))^2}{2b} \frac{dG(c)}{G(\hat{c})} - \frac{A}{b} + E, \tag{5}$$

where:

$$A = \frac{1}{n} + bp^* \quad \text{with } p^* = \int_0^{\hat{c}} p(c) \frac{G(c)}{G(\hat{c})} dc. \tag{6}$$

Once again the price aggregator A depends on both the mass of active firms n and the average price p^*, and the aggregate demand of each firm is provided by (1). Accordingly, a firm with marginal cost c obtains gross profits $\pi = (p-c)(A-bp)S$ by using price p, and thus it follows the price rule $p(c) = \frac{1}{2}(c+\frac{A}{b})$. The associated profits $\pi(c) = \frac{(A-bc)^2}{4b}S$ are positive for c below the cutoff $\hat{c} = \frac{A}{b}$.

This allows one to express the schedules for prices and profits as:

$$p(c) = \frac{c + \hat{c}}{2} \quad \text{and} \quad \pi(c) = \frac{b(\hat{c} - c)^2 S}{4}, \tag{7}$$

where the latter is a decreasing and convex function of the marginal cost. Entry takes place until the expected profits match the entry cost:

$$\int_0^{\hat{c}} \frac{b(\hat{c}-c)^2 S}{4} dG(c) = F. \tag{8}$$

Finally, by using (6), the price schedule in (7) and $A = b\hat{c}$, the mass of active firms can be expressed as:

$$n = \frac{2G(\hat{c})}{b\int_0^{\hat{c}}(\hat{c}-c)dG(c)} = \frac{2G(\hat{c})}{b\int_0^{\hat{c}} G(c)\, dc}, \tag{9}$$

where the last equality is obtained by integration by parts. In practice, (8) alone determines the equilibrium cutoff $\widehat{c}$, while (9) provides the mass of *ex-post* active firms. The measure N of entrant firms must be such that $n = NG(\hat{c})$.

A rise of market size S makes flatter the inverse demand and reduces the choke price, as suggested by Krugman *et al.* (2022: Ch. 8). The reduction of $\hat{c}$ follows immediately from (8), whose left hand side increases with respect to both S and $\hat{c}$. Formally, by differentiating (8), we get the elasticity:

$$\frac{d\ln\hat{c}}{d\ln S} = -\frac{\int_0^{\hat{c}}(\hat{c}-c)^2 dG(c)}{2\int_0^{\hat{c}}(\hat{c}-c)\hat{c}dG(c)} = -\frac{FN}{\hat{c}S} > -\frac{1}{2}.$$

Accordingly, a larger market size benefits firms whose marginal cost is sufficiently small, generating a selection effect ($\frac{d\hat{c}}{dS} < 0$): larger markets activate a smaller fraction of the entrant firms inducing the most inefficient ones to become inactive.[5]

As a consequence of the choke price reduction, the price schedule in (7) shows that an increase in market size generates also a competitive effect ($\frac{dp(c)}{dS} = \frac{\partial p(c)}{\partial\hat{c}}\frac{d\hat{c}}{dS} < 0$): larger markets reduce the prices of all active firms.

[5] Since the impact of market size on profits from (7) is:

$$\frac{d\ln\pi(c)}{d\ln S} = 1 + \frac{2\hat{c}}{\hat{c}-c}\frac{d\ln\hat{c}}{d\ln S},$$

there must be a threshold $\widetilde{c} \in (0,\hat{c})$ such that profits increase with market size for firms with $c \in [0,\widetilde{c})$ and decrease for $c \in (\widetilde{c},\hat{c}]$. Moreover, the production level decreases when the marginal cost is above a threshold larger than $\widetilde{c}$.

Finally, we can verify that a market size expansion attracts more firms, though this does not necessarily lead to more active firms *ex-post*: larger markets can either increase or reduce the mass of consumed varieties. From (9) we obtain the mass of entrants:

$$N = \frac{2}{b\int_0^{\hat{c}} G(c)\, dc} \quad \text{with} \quad \frac{\partial \ln N}{\partial \ln \hat{c}} = -\frac{nb\hat{c}}{2} < 0,$$

and:

$$0 < \frac{d \ln N}{d \ln S} = \frac{\partial \ln N}{\partial \ln \hat{c}} \frac{d \ln \hat{c}}{d \ln S} = \frac{2F}{bS} \frac{G(\hat{c})}{\left[\int_0^{\hat{c}} (\hat{c} - c) dG(c)\right]^2} < 1, \tag{10}$$

where last inequality follows from (8) and Jensen's inequality. However, the last expression in (9) for the mass of consumed varieties implies that $\frac{dn}{dS} = \frac{\partial n}{\partial \hat{c}} \frac{d\hat{c}}{dS} > 0$ if and only if $\int_0^{\hat{c}} \frac{G(c)}{G(\hat{c})} dc$ increases with respect to $\hat{c}$, a property that commonly used distributions, but not all, satisfy.

We can now use the partial derivatives of (5) with respect to the price schedule $p(c)$, the measure of active firms n and the cutoff $\hat{c}$, composed with the total derivatives of these variables with respect to S, to distinguish the welfare gains from lower prices, the gains from more varieties (which in principle might be negative), and the gains from selection of more efficient firms (which increases the average consumer surplus obtained by each variety).

In particular:

$$\frac{dV}{dS} = \underbrace{\frac{\partial V}{\partial p(c)} \frac{dp(c)}{dS}}_{competition} + \underbrace{\frac{\partial V}{\partial n} \frac{dn}{dS}}_{variety} + \underbrace{\frac{\partial V}{\partial \hat{c}} \frac{d\hat{c}}{dS}}_{selection} .$$

By using:

$$\frac{\partial V}{\partial p(c)} = -1, \quad \frac{\partial V}{\partial n} = \frac{F}{2SG(\hat{c})} \quad \text{and} \quad \frac{\partial V}{\partial \hat{c}} = -\frac{Ng(\hat{c})F}{2SG(\hat{c})},$$

we obtain:

$$\frac{\partial V}{\partial p(c)} \frac{dp(c)}{dS} = \frac{FN}{2S^2},$$

$$\frac{\partial V}{\partial n}\frac{dn}{dS} = \frac{FN}{2S^2}\left(-\frac{g(\hat{c})}{G(\hat{c})}\frac{FN}{S} + \frac{d\ln N}{d\ln S}\right),$$
$$\text{and } \frac{\partial V}{\partial \hat{c}}\frac{d\hat{c}}{dS} = \frac{F^2N^2}{2S^3}\frac{g(\widehat{c})}{G(\widehat{c})}.$$

Accordingly, (10) implies that also in case of heterogeneous firms the welfare gains from the competitive effects dominate the others: namely, $\frac{\partial V}{\partial p(c)}\frac{dp(c)}{dS} > \frac{\partial V}{\partial n}\frac{dn}{dS} + \frac{\partial V}{\partial \hat{c}}\frac{d\hat{c}}{dS}$. Moreover, the selection effects can be stronger than the variety effects even when the latter are positive, as illustrated by the following example.

Consider the case of a Pareto distribution with:

$$G(c) = \left(\frac{c}{\bar{c}}\right)^{\kappa} \quad \text{for } \kappa \geqslant 1,$$

and $g(c) = \frac{\kappa}{c}G(c)$. Then, the equilibrium mass of active firms and the cutoff can be computed from (9) and (8) as:

$$n = \frac{2(1+\kappa)}{b\hat{c}},$$

and:

$$\hat{c} = \left(\frac{2(1+\kappa)(2+\kappa)\bar{c}^{\kappa}F}{bS}\right)^{\frac{1}{2+\kappa}}.$$

A larger market size selects more efficient firms ($\frac{d\ln\hat{c}}{d\ln S} = -\frac{1}{2+\kappa}$) and attracts more firms, but in this case, it also activates a larger mass of firms ($\frac{d\ln n}{d\ln S} = \frac{1}{2+\kappa}$), generating positive gains from variety. By computing $\frac{\partial V}{\partial n}\frac{dn}{dS} = \frac{FN}{2S^2}\frac{1}{2+\kappa}$ and $\frac{\partial V}{\partial \hat{c}}\frac{d\hat{c}}{dS} = \frac{FN}{2S^2}\frac{\kappa}{2+\kappa}$, we finally obtain the following decomposition of the marginal benefit of market size:

$$\frac{dV}{dS} = \frac{(3+2\kappa)FN}{2(2+\kappa)S^2}\left[\underbrace{\frac{2+\kappa}{3+2\kappa}}_{competition} + \underbrace{\frac{1}{3+2\kappa}}_{variety} + \underbrace{\frac{\kappa}{3+2\kappa}}_{selection}\right].$$

Depending on the value of $\kappa \in [1,\infty)$, the competitive gains represent between 50% and 80% of the total welfare gains, the gains from variety no more than 20%

and the gains from selection between 20% and 50% of the total. For instance, a typical calibration value $\kappa = 5$ for trade models implies that 54% of the gains are due to competitive effects, 38% to selection effects and 8% to variety effects. The relative importance of the gains from selection increases with κ, while the relative importance of the other gains decrease with κ.

3 Conclusion

In this note we have provided a representative consumer foundation for the demand of Krugman *et al.* (2022: Ch. 8), displaying love for variety, and used it to measure the welfare impact of market size. While under homogenous firms the monopolistic competition equilibrium remains isomorphic to the one of Salop (1979), our formulation allows us to study the case of firm heterogeneity as well, and other applications to trade and industrial organization appear feasible.

References

Bertoletti, Paolo and Federico Etro, 2024, Insufficient Entry with Monopolistic Competition, DEMS WP 543, University of Milan-Bicocca.

Krugman, Paul, Maurice Obstfeld and Marc Melitz, 2022, *International Economics: Theory and Policy*, 12th Edition, Pearson.

Melitz, Marc, 2003, The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, *Econometrica*, 71, 6, 1695-725.

Melitz, Marc and Gianmarco Ottaviano, 2008, Market Size, Trade, and Productivity, *Review of Economic Studies*, 75, 1, 295-316.

Salop, Steven, 1979, Monopolistic Competition with Outside Goods, *Bell Journal of Economics*, 10, 1, 141-156.