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Bank Lending Policies and Green Transition

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Abstract

We consider a green monetary policy framework implemented by the central bank. Under this framework, firms and commercial banks decide whether or not to apply a green (environmentally friendly) or brown (conventional) investment and policy, respectively. We develop an evolutionary game to study the conditions under which a stable or unstable equilibrium is reached. If the green firms' revenues minus their bank loans and their transition costs are strictly greater than the brown firms' revenues and their pollution costs, together with (primary or subsidized) green interest rates such that the default risk is lower for green firms compared to brown ones, then the economy evolves to a asymptotically stable green state. In the green state all banks give green loans and all firms invest in green investment. If the condition is reversed the economy converges to a brown state. If the banks and the firms are indifferent towards the green and brown policy and investment respectively, the economy fluctuates from green to brown state. There may be multiple equilibria. Through a transcritical bifurcation we show how stability (instability) of the equilibria changes with the parameters.

Keywords: Climate Change; Evolutionary Dynamics; Green monetary policies; Firms Pollution
JEL classification: C70, C72, D21, K42, L21.

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1. Introduction

In December 2017, eight central banks and supervisors committed to establishing a Network of Central Banks and Supervisors to Green the Financial System (NGFS).[1] In 2018, NGFS members stated that '*...climate-related risks are a source of financial risk. It is therefore within the mandates of central banks and supervisors to ensure the financial system is resilient to these risks*'. Recently, Christine Lagarde commented that '*If we do not account for the impact of climate change on our economy, we risk missing a crucial part of the overall picture*' Lagarde (2022).[2] Indeed, the stability of the economy and of the financial system is affected by climate change that becomes an important source of structural transformation through two channels EBF (2017).[3] The first channel refers to physical risks that involve financial intermediaries or commercial banks both directly (e.g. damage to a bank's branches) and indirectly (affecting bank's customers, who may then default on their obligations, with possible losses for the bank). As a consequence, commercial banks could decide to reduce lending to firms in the most vulnerable areas, which could also have a negative impact on the monetary policy transmission channel. As for the second channel, it is related to the transition risks associated to an ungoverned transition to a low-carbon economy that could suddenly lower the price of energy assets associated with the extraction, conversion and use of fossil fuels. As a consequence, a race to liquidate the shares of utilities could arise, with implications for the global economic growth trajectory and the creation of stranded assets. The latter refers to the collapse of expectations of returns on invested capital due to the ungoverned transition, with a reduction of wealth for the end-owners of these stocks. Moreover, further risks may be transmitted indirectly to other economic agents through deeply interconnected financial markets (Hansen (2022), Kalkuhl *et al.* (2020), Semieniuk *et al.* (2022)).

[1] See: https://www.ngfs.net/en

[2] See: https://www.ecb.europa.eu/press/blog/date/2022/html/ecb.blog221107~1dd017c80d.en.html

[3] See: https://www.ebf.eu/ebf-media-centre/towards-a-green-finance-framework/, https://www.ngfs.net/sites/default/files/medias/documents/ngfs_first_comprehensive_report_-_17042019_0.pdf

For example, it has been shown that if the green transition is carried out in an orderly manner and policies are implemented immediately, emissions prices increase gradually. Indeed, on the one side firms adapt their business model and develop green technologies, and on the other, households change their consumption habits accordingly. The analysis performed by Alogoskoufis *et al.* (2021) suggests that banks' vulnerabilities to physical and transition risk are related to their exposures to manufacturing firms, even if this mainly holds for fragile banks located in particular areas.[4]

For these reasons also, the transition to green is necessary and inevitable, and requires massive public and private investments. Indeed, this overall investment in the EU for the 2021-30 years has been estimated at €466 billion on average per year (without considering the transport sector). Moreover, the new program REPowerEU has increased the investment needs of other €33 billion each year to improve Europe's energy sector diversification [5] However, such a transition might follow different dynamics (Dawid *et al.* (2021)), and also different pathways. In this perspective, a recent ECB paper by Emambakhsh *et al.* (2023) evaluates the implications for the real and the financial markets of three alternative transition paths (Accelerated, Late-push and Delayed transition) from 2023 to 2030, combining the transition paths developed by the NGFS with updated macroeconomic scenarios to fully consider energy-related developments. Findings suggest that costs of procrastination are high. An accelerated transition would bring considerable advantages to the euro area by preserving the optimal net-zero emissions path and containing financial risk. Among other findings, the authors show that EA banks' median portfolio probability of default (PD) depends closely on the transition pathway: in particular, under a late-push case, credit risk would grow until 2030 because of transition risk.

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[4] See: https://www.ecb.europa.eu/pub/financial-stability/fsr/special/html/ecb.fsrart202105_02~d05518fc6b.en.html, https://www.ecb.europa.eu/pub/pdf/other/ecb.climateriskfinancialstability202107~87822fae81.en.pdf

[5] Investing in Europe's future: The case for a rethink. Speech by Fabio Panetta, Member of the Executive Board of the ECB, at Istituto per gli Studi di Politica Internazionale (ISPI). See: https://www.ecb.europa.eu/press/key/date/2022/html/ecb.sp221111~9dfd501542.en.html

Thus, financial regulators such as central banks may induce the commercial banks to finance environmentally friendly firm projects with easier credit access to capital, i.e. special interest rates for green productive investment activities Muller (2021). Green credits, such as loans to projects that offer energy savings or emission reductions, are now driven by financial intermediaries addressing the green funding demands of the private sector and providing credit to businesses, and households (Beck & Demirguc-Kunt (2006); Park & Kim (2020); Yao *et al.* (2021)).

Recent empirical evidence (Braga *et al.* (2021); Liebich *et al.* (2023); Papoutsi *et al.* (2021)) shows that the Eurosystem holdings under the CSPP are biased towards more carbon-intensive companies, which have greater funding requirements and therefore constitute a higher proportion of total investments. Thus, as monetary policy should aim for 'market neutrality' so that bond purchases should be proportional to bonds outstanding, there is room to implement a green CSPP (Braga *et al.* (2021); Liebich *et al.* (2023); Bacchiocchi *et al.* (2024a)); Papoutsi *et al.* (2021) or a Green QE (Ferrari & Nispi Landi (2023)). Indeed, changing eligibility criteria define the the firm cost of liquidity. Particularly, Bacchiocchi *et al.* (2024a) state that in the face of a temporary green QE shock or the presence of green monetary policies, it can be shown that if the central bank increases its participation in green bonds, keeping total assets constant, the interest rate paid by green (brown) firms decreases (increases).

In this direction, a new specific question was included in the July 2023 survey of bank lending in the euro area (ECB, 2023) to measure the impact of climate change on bank lending to firms.[6] Firms are divided between green firms, firms in transition, and brown firms. Figure 1 shows that between 2022 and 2023, credit standards have tightened significantly due

[6] The euro area bank lending survey (BLS) was launched by the Eurosystem in 2003. More details: Köhler-Ulbrich, P., Hempell, H. and Scopel, S., "The euro area bank lending survey", Occasional Paper Series, No 179, ECB, 2016; and Burlon, L., Dimou, M., Drahonsky, A. and Köhler-Ulbrich, P., "What does the bank lending survey tell us about credit conditions for euro area firms?", Economic Bulletin, Issue 8, ECB, December 2019. See: `https://www.ecb.europa.eu/stats/ecb_surveys/bank_lending_survey/html/index.en.html`, `https://www.ecb.europa.eu/press/pr/date/2023/html/ecb.pr231024~c42cea39db.en.html`

to climate risks for loans to brown businesses, while climate change contributed to improve credit conditions for lending to green and transition businesses. Similarly to credit standards, commercial banks' effective lending requirements to brown firms tightened while those to green and transition firms loosened. Interestingly, green and transition firms augmented their demand of new loans for fixed investments and corporate restructuring for climate change purposes, mainly referred to climate risks was fixed investment and corporate restructuring. In contrast, brown firms decreased loan demand for climate risks considerations.

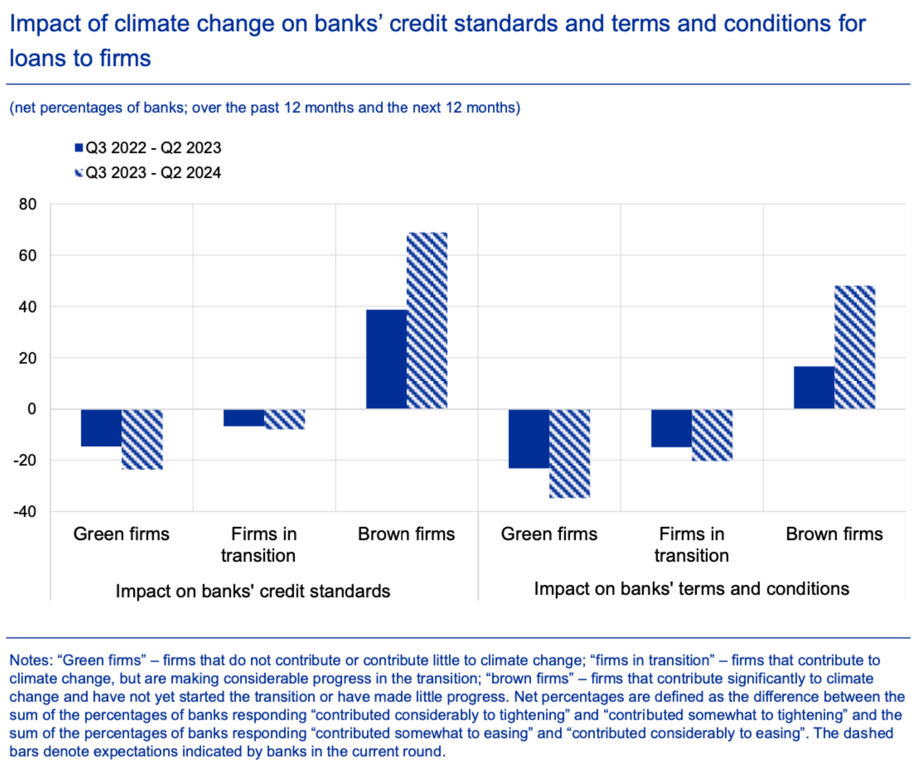

Figure 1: This chart is from the July 2023 Bank Lending Survey (BLS), Chart 21.

Notwithstanding this, during the last year the ECB's monetary policy tightened in response to the high inflation rates. The increase in interest rates improve the costs of financing investments, especially those for green technologies. Thus, the pace of decarbonisation may slow down as capital costs become larger (Acevedo *et al.* (2021); Hagspiel *et al.* (2021)).[7]

[7]More information in Isabel Schnabel's Speech, Member of the Executive Board of the ECB, at the Inter-

Therefore, under the previous premises as empirical motivation, in this research paper, we study the interplay between banks and firms according to the decision whether or not to opt for green policies. To this aim, we analyze the strategic coordination between banking lending policies, that is, special interest rates and firms' green productive decisions. The development of our game-theoretical modeling follows the baseline approach by Bai & Lin (2024), Dawid *et al.* (2021), and Ferrari & Nispi Landi (2023). We augment these models by analysing the strategic interaction between banks and firms using an evolutionary game model. In our setup, commercial banks must decide whether or not to apply a green policy, and for their part, firms must decide whether or not to invest in environmentally friendly technologies, which are being characterized as green or brown firms. We analyze the main parameters to converge in an evolutionary stable path towards greening or not, namely the role of the interest rates, transition costs, polluting costs, and default probabilities. We show that the green state (i.e., banks and firms opt for the green strategy) is asymptotically stable if the green firms' revenues minus their bank loans and their transition costs are greater than the brown firms' revenues and their pollution costs, together with (primary or subsidized) green interest rates such that the default risk is lower for green firms compared to brown ones. Otherwise, the economy converges to the conventional situation of the browning economy. Moreover, our model allows for cases of degenerate bifurcations. The latter happens when the green interest rates or transition costs are high enough so that the segment of fixed points towards the brown-level equilibrium enlarges. This is a key finding that suggests, on the one side that monetary policy matters for the transition to green, and on the other, that innovation is also an important factor in the measure it reduces these transition costs. Finally, we show that the transition to green might be realized regardless of green monetary policy, e.g., in the presence of high carbon tax.

national Symposium on Central Bank Independence, Sveriges Riksbank, Stockhol: Monetary policy tightening and the green transition. *International Symposium on Central Bank Independence.* 10th January 2023, Stockholm. https://www.ecb.europa.eu/press/key/date/2023/html/ecb.sp230110~21c89bef1b.en.html

The remainder of the paper is organized as follows. Section 2 introduces the model (a one-shot game) and the dynamic strategies (the evolutionary game). While section 3 analyzes the equilibrium stability conditions and the main implications. Section 4 concludes the paper.

2. Model setup

In this section, we present an evolutionary game between banks (meant by LB) and firms (indicated by F). Banks must decide whether or not to apply a green policy; for their part, firms must decide whether or not to invest in environmentally-friendly technologies, that are being characterized as green or brown firms, labeled as $g-$firms, and $b-$firms, respectively. Particularly, we consider that the green firms face a fixed cost of productive activities by environmentally-friendly investment through bank loans, $L > 0$, charged at an interest rate $r_j > 0 \quad \forall j = g, b$; depending on whether the banking policy is green, that is, a green premium where the green interest rate (primary or subsidized) is low r_g; or else, if the banking policy is conventional, the loan is obtained at a high-interest rate $r_b \geq r_g$. That is, such a differential in interest rates depends on whether the banks are proactive toward green activities or not (Papoutsi *et al.* (2021); Bacchiocchi *et al.* (2024a); Liebich *et al.* (2023); Kumar *et al.* (2023); Ferrari & Nispi Landi (2023)). Furthermore, the investment costs of green firms are determined by $L(r_j + \gamma_j) > 0$, which represents the cost of investing in green technologies, facing green or brown interest rates (r_j), and also considering a productive process of transition to green, $\gamma_j > 0 \ \forall j = g, b$; this transition differs depending on whether it is dealing with a green or brown bank policy. That is, γ_g is the productive transition to green when facing a green bank policy, while γ_b is the transition when facing a brown bank policy, $\gamma_g \neq \gamma_b$. This parameter γ_j is also related to the level of innovation that characterizes the environment in which firms operate. Indeed, innovation is at the heart of the transition to a cleaner global environment (OECD (2019) OECD, 2019). This includes not only technological innovation but also innovation in economic and social systems and lifestyles. Innovation is the main source of modern economic growth, which implies that the green transition is compatible not only with long-term economic growth but also with a vast range of economic opportunities for businesses. Thus, the deeper the innovation, the lower the cost of transition to green, the lower γ_j. In contrast, polluting (brown) $b-$firms face the $r_j L$ bank loan costs, polluting production costs, measured by $C > 0$, and a carbon tax τC, with $\tau \in (0, 1)$.

Banks, for their part, make profits through the returns generated by loans, $L > 0$, such loans granted to green and brown firms with interest rates r_g and r_b. Banks provide loans to firms, but when the bank implements a green policy, a lower interest rate r_g applies; instead, when the bank implements a conventional policy, the higher interest rate r_b applies, regardless of the investment type.[8]

The financial structure of firms matters, and there are degrees of risk in loan losses Stiglitz (2010)), that is the so-called non-performing loans. These default probabilities are measured by four different parameters given by $\alpha_g, \alpha_b, \beta_g, \beta_b$. The parameter $\alpha_j \ \ \forall j = g, b$ measures the default probability of green firms, but this risk is different if the loan is granted by a green or conventional bank, which is specified by the subindex, g, b. That is, $\alpha_g \in (0, 1)$ is the risk of bad loans when a green bank meets a green firm, while $\alpha_b \in (0, 1)$ is when a brown bank meets a green firm. The parameter $\beta_j \ \ \forall j = g, b$ measures the default probability of brown firms. That is, $\beta_g \in (0, 1)$ is when a green bank meets a brown firm, and $\beta_b \in (0, 1)$ is when a conventional bank meets a brown firm. Under a green monetary policy scheme, we can assume that the probability of default is the lowest in the couple green firms together with green banks, that is, $\alpha_g \leq \alpha_b$, $\alpha_b \leq (\beta_g, \beta_b)$. This assumption is related to the green finance paradigm, according to which assets with higher sustainability scores are, in the long run, more resilient to shocks and, as a result, can provide better combinations of risk and return, benefiting investors and society as a whole (Bernardini *et al.* (2021)).

[8]Here we do not consider the phenomenon of green-washing, in which some firms issue green bonds to bear a lower financing cost, without investing in green technologies.

Therefore, the game between the decisions of banking policy, LB, and the decisions of the firms F about greening or not (G, B, respectively), is in the following normal-form representation, i.e. the payoff matrix:

F / LB	G	B
G	$\pi_g - (r_g + \gamma_g)L, \quad r_g(1-\alpha_g)L - \alpha_g L$	$\pi_g - (r_b + \gamma_b)L, \quad r_b(1-\alpha_b)L - \alpha_b L$
B	$\pi_b - r_g L - (1+\tau)C, \quad r_g(1-\beta_g)L - \beta_g L$	$\pi_b - r_b L - (1+\tau)C, \quad r_b(1-\beta_b)L - \beta_b L$

where in each cell, the first value refers to the firms payoff and the second to the bank's payoffs, and $\pi_g \geq 0$ measures the revenues (gross payoff) of the green firm and $\pi_b \geq 0$ is the revenue of the brown firm. Such revenues are related to consumers' attitudes towards green or brown products. The larger the consumers' attitude, the larger the revenues, all else equal.

To compute the expected payoffs, we must consider the probabilities for each economic agent to meet its counterpart, following their adopted strategies. Let us define the mixed-strategy simplex:

$$\Delta^{LB} = \left\{ x \in R_+^2 : x_g + x_b = 1 \right\}$$

which represents the distribution of the banks profile $x = (x_g,\ x_b) \in \Delta^{LB}$, i.e. x_g represents the share of green banks, while x_b represents the share of brown banks. At the same time, the mixed-strategy simplex:

$$\Delta^{F} = \left\{ y \in R_+^2 : y_g + y_b = 1 \right\}$$

is representing the distribution of the firms' profile $y = (y_g,\ y_b) \in \Delta^F$ where y_g represents the share of green firms, and y_b is the share of brown firms. So, with these distribution profiles, which are probabilities that an agent adopts one strategy or another, we can get the agents' (firms and banks) expected payoffs.

The $g-$firm expected payoff is given by:

$$E(F_g) = x_g(\pi_g - (r_g + \gamma_g)L) + (1 - x_g)(\pi_g - (r_b + \gamma_b)L) \tag{1}$$

and for a $b-$firm is:

$$E(F_b) = x_g(\pi_b - r_g L - (1 + \tau)C) + (1 - x_g)(\pi_b - r_b L - (1 + \tau)C) \tag{2}$$

By comparing the expected payoffs, we can deduce the strategy that each economic agent will want to choose. Firms prefer greening if $E(F_g) > E(F_b)$, and this happens if the share of banks' greening policy is large enough, that is,

$$x_g^* > \bar{x}_g = \frac{\pi_g - \pi_b - L\gamma_b + C(1 + \tau)}{L(\gamma_g - \gamma_b)} \tag{3}$$

therefore, green firms prevail if such threshold value approaches zero, $\bar{x}_g \to 0$, since most banks opt for green policies. From (3) we can deduce the conditions to overcome such a threshold value, that is

- The profits of green firms is $\pi_g = \pi_b - C(1+\tau) + L\gamma_b$, which means that such profits of green firms must be as large as the net profits of brown firms $(\pi_b - C(1+\tau))$, but also considering the transition to green production when banking policy is brown ($L\gamma_b$, the worst scenario for the $g-$firms).

- The productive transition to green when monetary policy is brown is defined by $\gamma_b = \frac{\pi_g - \pi_b + C(1+\tau)}{L}$, which means such productive transition to green must be equal to the difference between the gross profits of the green and brown firms plus production cost and the payment of the carbon tax by the polluting firms, all this in relation to the bank loan size.

- Similarly, the loans level is $L = \frac{\pi_g - \pi_b + C(1+\tau)}{\gamma_b}$, which means that such loans must be an amount determined by the difference in firms profits, pollution costs, and all this in relation to the productive transition to green when banking policy is brown.

- The carbon tax value is large enough, $\tau = \frac{\pi_b - \pi_g - C + L\gamma_b}{C}$ which means that staying brown is costly enough for firms due to tax imposition.

- The polluting production costs are $C = \frac{\pi_b - \pi_g + L\gamma_b}{1+\tau}$, which means that brown technologies must be sufficiently inefficient.

Similarly, the expected payoff for the green banking policy is given by:

$$E\left(LB_g\right) = y_g\left(r_g(1-\alpha_g)L - \alpha_g L\right) + (1-y_g)\left(r_g(1-\beta_g)L - \beta_g L\right) \tag{4}$$

and the expected payoff of the conventional bank policy is:

$$E\left(LB_b\right) = y_g\left(r_b(1-\alpha_b)L - \alpha_b L\right) + (1-y_g)\left(r_b(1-\beta_b)L - \beta_b L\right) \tag{5}$$

Therefore, banks prefer to apply a green monetary policy if $E\left(x_g|y\right) > E\left(x_b|y\right)$, and it happens if the share of $g-$firms is large enough, i.e.

$$y_g^* > \bar{y}_g = \frac{r_g(1-\beta_g) - r_b(1-\beta_b) + \beta_b - \beta_g}{r_g(\alpha_g - \beta_g) + r_b(\beta_b - \alpha_b) + \alpha_g - \alpha_b + \beta_b - \beta_g}, \tag{6}$$

therefore, green monetary policies prevail if such threshold value approaches zero, $\bar{y}_g \to 0$. Therefore, the conditions to overcome such a threshold (6) are given by:

- On the one hand, the green interest rate is defined by $r_g = \frac{r_b(1-\beta_b) - \beta_b + \beta_g}{1-\beta_g}$, which means that such an interest rate must be compared to the brown interest rate and the default probabilities of brown firms. Note that the higher the default risk of brown firms under a green bank policy scheme, the higher the green interest rate.

- On the other hand, the brown interest rate is defined by $r_b = \frac{r_g(1-\beta_g) - \beta_g + \beta_b}{1-\beta_b}$, which means that the profitability of implementing a brown interest rate is compared to the profitability of implementing a green interest rate considering the difference in default of brown firms under brown and green banking schemes. In this circumstance, the greater the default probability of the brown firm under a brown banking scheme, the higher the brown interest rate.

This game can be expressed as a coordination game with two pure Nash equilibria:

$$(G, G) = ((1,\ 0)\,;(1,\ 0)) \text{ if } \pi_g > \pi_b > 0 \text{ y}, \alpha_g < \beta_b \text{ and } (B, B) = ((0,\ 1)\,;(0,\ 1)) \text{ otherwise},$$

There exists a Nash equilibrium in mixed strategies $(\bar{x}_g, \bar{y}_g)$ defined by (3) and (6), such that the players are indifferent between the pair of strategies G and B. Let's now move on to analyzing the evolutionarily dynamic game. So the actors, banks and firms, interact dynamically to select their decisions on whether or not to opt for a green transition. Thus indicating the different equilibrium situations and offering conditions of stability.

2.1. Dynamic strategies

We have analyzed the game in its static form; let us now analyze this game as an evolutionary dynamic process. The term evolutionary dynamic means that the more successful strategies G or B tend to proliferate. In contrast, the less successful tend to disappear, and so to explicitly model a selection process, specifying how population shares associated with different strategies in a game evolve. Recall that total populations of Banks (LB) and Firms (F) are normalized, i.e. $(x_g + x_b) = 1$ and $(y_g + y_b) = 1$. That is, x_j and y_j denote the fractions of Banks and Firms, respectively, choosing the j-strategy ($j \in \{G, B\}$). Then, the corresponding average payoffs by equations by equations (4)-(5):

$$\begin{aligned}\bar{E}_{LB} = x_g\,(E(LB_g)) + (1 - x_g)\,(E(LB_b)) = L x_g\,[r_g - y_g \alpha_g (1 + r_g) - (1 - y_g)(1 + r_g)\beta_g] \\ -L(1 - x_g)\,[y_g \alpha_b (1 + r_b) - r_b + (1 - y_g)(1 + r_b)\beta_b]\end{aligned} \tag{7}$$

and by equations (1)-(2),

$$
\begin{aligned}
\bar{E}_F = y_g\left(E(F_g)\right) + (1-y_g)\left(E(F_b)\right) = \pi_g y_g + \pi_b(1-y_g) - C(1-y_g)(1+\tau) \\
-L\left[(1-x_g)r_b + (1-x_g)y_g\gamma_b + x_g r_g + x_g y_g \gamma_g\right]
\end{aligned}
\tag{8}
$$

Given the above expected payoffs (eqs. (1), (2), (4), (5) and (7), (8)), where in order to simplify the notations we define $x = x_g$ and $y = y_g$ so that $x_b = 1 - x$ and $y_b = 1 - y$, we can work with the well-known Replicator Dynamics (RD) in continuous time which is represented by the following system of two differential equations,[9]

$$
\begin{cases}
\dot{x} = x\left[E\left(LB_g\right) - \bar{E}_{LB}\right] = x(1-x)L\left[r_b(y(\alpha_b-\beta_b)+\beta_b-1) - r_g(y(\alpha_g-\beta_g)+\beta_g-1) - \right. \\
\qquad \left. -y(\alpha_g-\alpha_b+\beta_b-\beta_g)+\beta_b-\beta_g\right] \\
\\
\dot{y} = y\left[E\left(F_g\right) - \bar{E}_F\right] = y(1-y)\left[\pi_g - \pi_b + C(1+\tau) - L\gamma_g x - L\gamma_b(1-x)\right]
\end{cases}
\tag{9}
$$

This RD system (9) is a nonlinear two-dimensional dynamical system in continuous time. The replicator dynamics states that the number of agents adopting a given strategy grows if the expected payoff of that strategy is greater than the average payoff attained by the population they belong to. Otherwise, it decreases. In other words, the proportion of agents using a dynamic strategy G increases if its payoff is more significant than the average payoff of the corresponding population. The first step to shed some light on its qualitative dynamic

[9]The mathematical formulation of the replicator dynamics is due to Peter D. Taylor and Leo Jonker (1978). They imagine a large population of agents who are randomly matched over time to play a finite symmetric two-player game, just as in the setting for evolutionary stability. Studying the replicator dynamics of a system is an increasingly utilized tool for understanding the evolution, for example, of institutions and social norms, providing many applications in economics. Taylor and Jonker in 1978 published their seminal paper, entitled "Evolutionarily stable strategies and game dynamics", in which the mainly adopted dynamic foundation of evolutionary dynamics is the one known as "replicator dynamics", explicitly expressed through differential or difference equations, thus recognizing the close links between a game-theoretic approach and the theory of dynamical systems. See, https://doi.org/10.1016/0025-5564(78)90077-9

behavior is the study of the existence of equilibrium points: their localization and local stability properties.

The equilibrium points of (9) are the solutions of the five degree algebraic system of two equations, i.e.

$$\begin{cases} (1-x)L\left[r_b(y(\alpha_b-\beta_b)+\beta_b-1)-r_g(y(\alpha_g-\beta_g)+\beta_g-1)-y(\alpha_g-\alpha_b+\beta_b-\beta_g)+\beta_b-\beta_g\right]=0 \\ y(1-y)\left[\pi_g-\pi_b+C(1+\tau)-L\gamma_g x-L\gamma_b(1-x)\right]=0 \end{cases}$$

So the RD (9) system, $(\dot{y},\ \dot{x})$, admits five steady states or dynamic equilibria, i.e. four corner equilibria $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$ and one interior $(\bar{x},\ \bar{y})$ equilibrium[10]:

$$\bar{x}=\frac{\pi_g-\pi_b-L\gamma_b+C(1+\tau)}{L(\gamma_g-\gamma_b)}, \qquad \bar{y}=\frac{r_g(1-\beta_g)-r_b(1-\beta_b)+\beta_b-\beta_g}{r_g(\alpha_g-\beta_g)+r_b(\beta_b-\alpha_b)+\alpha_g-\alpha_b+\beta_b-\beta_g}.$$

Overall, these equilibria can be interpreted as follows:

- Equilibrium $(0,0)$ is the brown-level equilibrium in which banks and firms are both choosing (B,B). In this situation there is no transition to green as neither firms nor banks find the green strategy profitable.

- Equilibria $(0,1)$ and $(1,0)$ are mismatches of profiles (B,G), (G,B). In these two equilibria, when firms choose G, banks choose B and vice versa, and there is no transition to green.

- Equilibrium $(1,1)$ is the green-level equilibrium in which banks and firms are (G,G), that is, the economy is experimenting with a full transition to green.

[10] In particular, this unique interior equilibrium $(\bar{x},\bar{y})$ is located at the intersection of the two nullclines:

$$y(\alpha_g-\alpha_b+\beta_b-\beta_g)+r_g\left(y(\alpha_g-\beta_g)+\beta_g-1\right)=\beta_g-\beta_b-r_b\left(y(\alpha_b-\beta_b)+\beta_b-1\right),$$

$$L\gamma_g x+L\gamma_b(1-x)=\pi_g-\pi_b+C(1+\tau)$$

and it is an equilibrium lying in the interior of the square $\mathcal{C}=[0,1]\times[0,1]$, such that $0<\bar{x}_g<1$ and $0<\bar{y}_g<1$.

- The interior equilibrium $(\bar{x}, \bar{y})$ has the separatrix role between the green equilibrium $(1,1)$ and the brown equilibrium $(0,0)$. This equilibrium could also be a center to which the system gravitates around. However, as we will demonstrate in the next section, this is never a stable equilibrium. Intuitively, a situation in which both profiles distributions, green and brown, of firms and banks coexist, will not hold in the long run.

3. Equilibrium stability conditions (green *vs* brown)

Below we summarize our main results: the steady-state stability conditions for the evolutionary dynamics of the dynamic greening strategies of banks and companies under a green monetary policy framework. The dynamic greening strategies represented by the RD system (9 of banks and firms yield the following conditions of stability of the stationary states (see the proof in appendix):[11]

1. The green-level equilibrium $(1,1)$ is an asymptotically stable point (or sink) if:

$$\pi_g - L\gamma_g > \pi_b - C(1+\tau), \quad r_g(1-\alpha_g) - \alpha_g > r_b(1-\alpha_b) - \alpha_b$$

while if these inequalities are reversed, then it is an unstable equilibrium. Therefore, the asymptotically stable green equilibrium is achieved when both the net profits of green firms (considering the cost of the green transition) are higher than the net profits of brown firms (considering pollution costs), and also when the profitability of green banks for applying green interest rates is higher than that of brown banks for applying brown interest rates (considering the default risks of green companies).

[11] Let us remember that by definition:

1. An asymptotically stable equilibrium is called stable node (or sink) if all of the eigenvalues have negative real part.
2. An equilibrium is an unstable node or source if all of the eigenvalues have positive real part.
3. An equilibrium is a saddle point if we have two real distinct eigenvalues of opposite sign.

2. The brown-level equilibrium $(0,0)$ is an asymptotically stable point (or sink) if:

$$\pi_b - C(1+\tau) > \pi_g - L\gamma_b, \quad r_b(1-\beta_b) - \beta_b > r_g(1-\beta_g) - \beta_g$$

while if these inequalities are reversed, then it is an unstable equilibrium. Obviously, this is the opposite case of the previous item; that is the asymptotic stability of the brown equilibrium is achieved when both the net profits of the brown firms are higher than the net profits of the green firms (when the monetary policy is brown) and also the profitability of the brown bank is greater than that of the green bank, given the default risks of brown firms.

3. Equilibria $(1,0)$ and $(0,1)$ are unstable nodes (or sources) (since: $r_g(1-\alpha_g) > r_b(1-\alpha_b)$ and $\pi_g + L\gamma_g > \pi_b + C(1+\tau)$). As it should be noted, when the equilibrium $(0,0)$ is unstable and is a source, then the equilibrium $(1,1)$ is the only asymptotically stable equilibrium (the interior equilibrium point disappears from the unit square); in the same way when the equilibrium $(1,1)$ is unstable (or nodal source) the equilibrium $(0,0)$ is the only asymptotically stable one. The corner equilibria $(0,1)$ and $(1,0)$ are always unstable. The following figures 2-3 simulate these results.

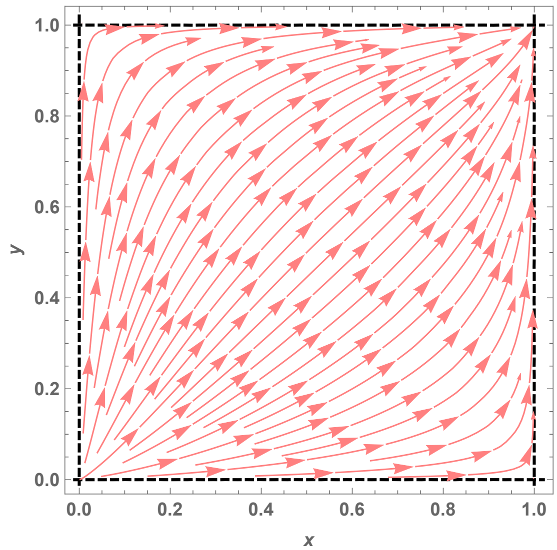

Figure 2: Phase portrait of the replicator system $(\dot{x}, \dot{y})$. The equilibrium point $(0, 0)$ is a source while the equilibrium $(1, 1)$ is an asymptotically stable point. The parameter configuration values are: $\pi_g = 2.2$, $\pi_b = 2$, $L = 1$, $C = 0.9$ $\tau = 0.05$, $\gamma_b = 0.5$, $\gamma_g = 0.3$, $r_g = 0.05$, $r_b = 0.06$, $\alpha_g = 0.14$, $\alpha_b = 0.2$, $\beta_g = 0.3$, $\beta_b = 0.32$.

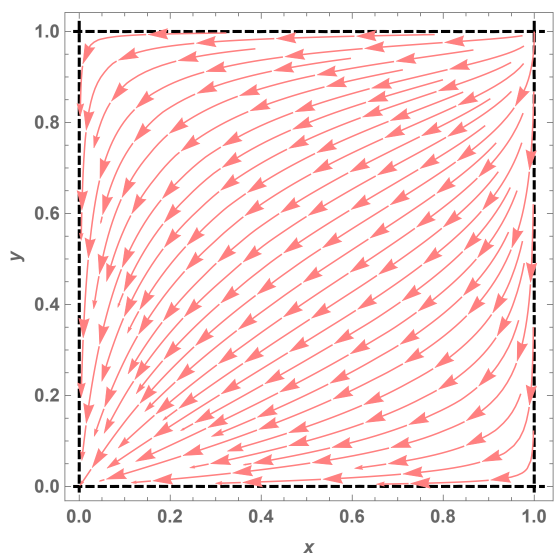

Figure 3: Phase portrait of the replicator system $(\dot{x}, \dot{y})$. The equilibrium point $(1, 1)$ is a source while the equilibrium $(0, 0)$ is an asymptotically stable point. The parameter configuration values are: $\pi_g = 1.2$, $\pi_b = 2$, $L = 1$, $C = 0.9$ $\tau = 0.05$, $\gamma_b = 0.6$, $\gamma_g = 0.6$, $r_g = 0.04$, $r_b = 0.05$, $\alpha_g = 0.14$, $\alpha_b = 0.2$, $\beta_g = 0.4$, $\beta_b = 0.32$.

4. Interesting is the case of interior equilibrium. This equilibrium $(\bar{x}, \bar{y})$ maybe a saddle point, i.e. a threshold since it separates the basins of attraction of the brown-level and

green-level equilibria (simulation results in Figure 4), if:

$$\pi_g > \pi_b - C(1+\tau), \quad \gamma_g < \gamma_b, \quad r_b\alpha_b > r_g\alpha_g, \quad r_b\beta_b > r_g\beta_g$$

Hence, the saddle point interior equilibrium exists when: (i) the green firms' revenues are greater than the brown firms' revenues considering the production costs and the carbon tax, (ii) the cost of transition to green is lower if the interest rate is green; (iii) the the brown interest rate times the green firms' probability of default facing a brown banking system is higher than the green interest rate times the green firms' probability of default facing a green banking scheme, but also (iv) the brown interest rate is higher than the green interest rate considering, respectively, the probability of default of brown firms facing green and brown banking schemes. Note that conditions (ii) and (iii) are generally satisfied given the model's assumptions.

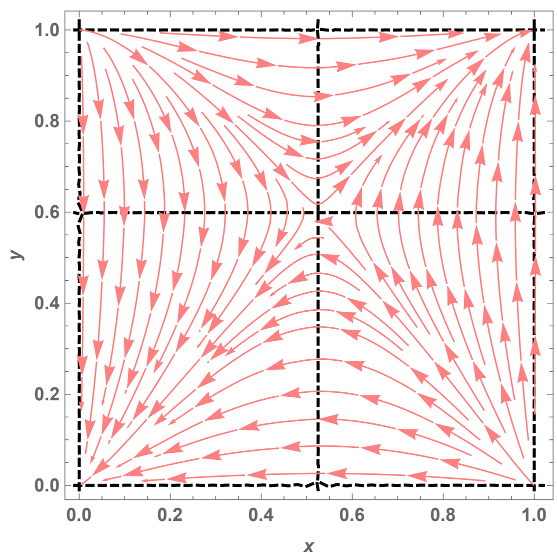

Figure 4: Phase portrait of the replicator system $(\dot{x}, \dot{y})$. The horizontal dashed line is the nullcline $\dot{x} = 0$ and the vertical dashed line is the nullcline $\dot{y} = 0$. The parameter configuration values are: $\pi_g = 2.6$, $\pi_b = 2.5$, $L = 1$, $C = 0.9$ $\tau = 0.05$, $\gamma_b = 1.6$, $\gamma_g = 1.3$, $r_g = 0.02$, $r_b = 0.04$, $\alpha_g = 0.1$, $\alpha_b = 0.19$, $\beta_g = 0.4$, $\beta_b = 0.2$.

Moreover, the interior equilibrium $(\bar{x}, \bar{y})$ may also be characterized by oscillations with fixed amplitude and therefore this fixed point is a center (simulation results in Figure

5), and this happens if:

$$\pi_g = \pi_b - C(1+\tau), \quad \gamma_g = \gamma_b, \quad r_b\alpha_b = r_g\alpha_g, \quad r_b\beta_b = r_g\beta_g$$

so this center-type equilibrium occurs when: (i) the profits of a green firm are equal to the net profits of a brown firm, (ii) the transition to green productivity under green or brown banking schemes are equal; and (iii) the green and brown interest rates considering the default risks of firms are also equal to each other. That is, this is a situation such that both banks and firms are continuously changing (fluctuating) from a green action to a brown action and vice versa, since there is total indifference between these actions on the part of the players, banks and firms.

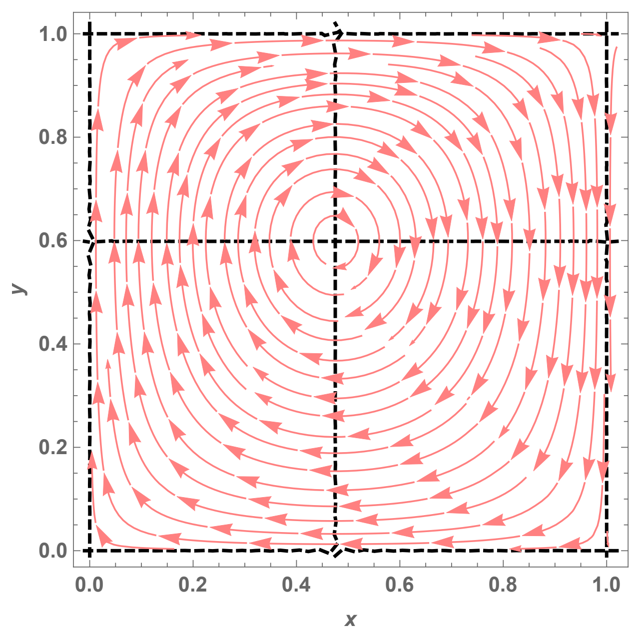

Figure 5: Phase portrait of the replicator system $(\dot{x}, \dot{y})$. The horizontal dashed line is the nullcline $\dot{x} = 0$ and the vertical dashed line is the nullcline $\dot{y} = 0$. The parameter configuration values are: $\pi_g = 1.65$, $\pi_b = 1.5$, $L = 1$, $C = 0.9$ $\tau = 0.05$, $\gamma_b = 0.5$, $\gamma_g = 2.5$, $r_g = 0.022$, $r_b = 0.023$, $\alpha_g = 0.24$, $\alpha_b = 0.4$, $\beta_g = 0.49$, $\beta_b = 0.29$.

Regarding the stability of the internal equilibrium we can affirm the following. Suppose $f_1 = \pi_g - (\pi_b + L\gamma_b - C(1+\tau))$, $f_2 = \pi_g - (\pi_b + L\gamma_g - C(1+\tau))$, $f_3 = r_g(1-\alpha_g) - r_b(1-\alpha_b) + \alpha_b - \alpha_g$, and $f_4 = r_g(1-\beta_g) - r_b(1-\beta_b) + \beta_b - \beta_g$. The interior equilibrium point

$(\bar{x}, \bar{y}) > 0$ can be rewritten as:

$$\bar{x} = \frac{f_1}{f_1 - f_2}, \qquad \bar{y} = \frac{f_4}{f_3 + f_4}$$

Suppose $\lambda_{1,2}$ are the eigenvalues for $(\bar{x}, \bar{y})$. Based on the characteristic polynomial of its Jacobian Matrix (see Appendix), $\lambda_{1,2}$ are roots of the quadratic equation:

$$\lambda^2 + L\bar{x}\bar{y}\left(r_g(1-\alpha_g) - r_b(1-\alpha_b) + \alpha_b - \alpha_g\right)\left(\pi_g - \pi_b + C(1+\tau) - L\gamma_g\right) = 0$$

$$\lambda^2 + \frac{f_1 f_2 f_3 f_4}{(f_1 - f_2) f_3 + f_4} = 0$$

1. If $f_2 f_3 < 0$ we obtain that $\lambda_{1,2}$ are real (one positive and one negative), so $(\bar{x}, \bar{y})$ is a saddle point.

2. If $f_2 f_3 > 0$ we obtain that $\lambda_{1,2}$ are purely imaginary, then $(\bar{x}, \bar{y})$ is a center point.

3. If $f_3 = 0$ we get $\bar{y} = 1$ and $\lambda_{1,2} = 0$, and we have $(x, 1)$ as manifold of equilibria. It means that along $y = 1$ we have infinitely many equilibria.

4. If $f_2 = 0$ we get $\bar{x} = 1$ and $\lambda_{1,2} = 0$, and we have $(1, y)$ as manifold of equilibria. It means that along $x = 1$ we have infinitely many equilibria.

Next, we analyze bifurcations as qualitative or topological changes in the phase portrait (i.e., the structure of the orbit) for fields and vector maps as some parameter varies, for example, the change in the stability of the internal fixed point and the appearance or disappearance of fixed points or periodic orbits (Kuznetsov (1998)).

3.1. Transcritical bifurcations

A bifurcation indicates a transition from one qualitative type of dynamic to another. In our case, this happens when the interior equilibrium coalesces at the border of the unitary square $\mathcal{C} = [0, 1] \times [0, 1]$. The most standard cases are the **transcritical bifurcations at the vertices**, and there are 4 conditions given by:

Condition 1 $\pi_g = \pi_b + L\gamma_b - C(1+\tau) \geq 0.$

Condition 2 $\pi_g = \pi_b + L\gamma_g - C(1+\tau) \geq 0.$

Condition 3 $r_g = \frac{r_b(1-\alpha_b)-\alpha_b+\alpha_g}{1-\alpha_g} \geq 0.$

Condition 4 $r_g = \frac{r_b(1-\beta_b)-\beta_b+\beta_g}{1-\beta_g} \geq 0.$

So, we can have 4 possibilities for some transcritical bifurcation to occur, i.e.

1. If condition 1 & 4 satisfied, the $(\bar{x}, \bar{y})$ coalesce with $(0,0)$.

2. If condition 2 & 4 satisfied, the $(\bar{x}, \bar{y})$ coalesce with $(1,0)$.

3. If condition 1 & 3 satisfied, the $(\bar{x}, \bar{y})$ coalesce with $(0,1)$.

4. If condition 2 & 3 satisfied, the $(\bar{x}, \bar{y})$ coalesce with $(1,1)$.

Obviously the *good-green bifurcation* is the one according to which all banks and firms are choosing green, $(1,1)$. This situation is given by the fulfillment of conditions 1 & 4 such that the interior equilibrium transposes with the equilibrium $(0,0)$ but this becomes a repulsor (see Figure 6), and thus the economy converges to equilibrium $(1,1)$

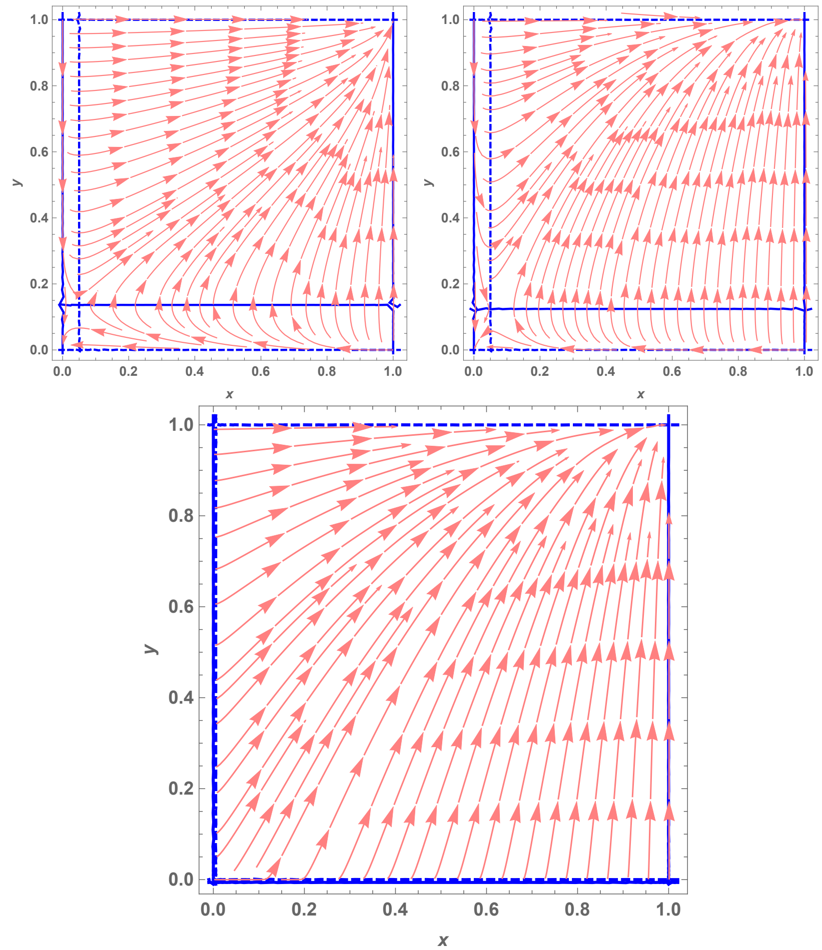

Figure 6: Phase portrait of the replicator system $(\dot{x}, \dot{y})$, where the numerical values of the parameter settings satisfy conditions 1 and 4.

That is, the figure 6 is obtained when conditions 1 & 4 hold. From the firms' perspective (condition 1), this means that the revenues of green firms π_g must be large enough to achieve the total profits of brown firms (i.e. profits minus production costs and carbon tax, $\pi_b - C(1+\tau)$) and transition costs to green when bank lending policy is brown (i.e. worst case scenario for green firms), $L\gamma_b$. But also, from the perspective of the banks (condition 4) it is such that the green interest rate (primary or subsidized) r_g is measured according to the brown interest rate, r_b, and the default risks of brown firms when brown banks lend to brown firms, β_b. While such a green interest rate, r_g, is a decreasing function of non-performing loan risk when the green bank lends to brown firms, β_g.

Next, we also get a manifold of equilibria. It means that along $y = 1$ or $x = 1$ we have infinitely many equilibrium. This can happen every time the interior equilibrium point merges and exchanges ending on one of the edges of the unit square, since on that edge a line segment of fixed equilibrium points is achieved, where at these fixed points there is stability, instability, or indeterminacy. In particular, the following interesting cases may occur.

1. The interior equilibrium coalesces at the upper-edge of the unitary square (see Figure 7), which means that under certain conditions, the economy is characterized by a large variety of equilibria where there is a large variety of banks, either brown or green, but above a certain threshold all firms are green, or else if not exceeded such a threshold, then the economy converges towards the stable steady state of brown banks and brown firms. That is, a bifurcation $((\bar{x}, \bar{y}) = (\bar{x}, 1))$ occurs if any of the following conditions are met:

$$r_g = \frac{r_b(\alpha_b - 1) + \alpha_b - \alpha_g}{\alpha_g - 1}, \quad r_b = \frac{r_g(\alpha_g - 1) + \alpha_g - \alpha_b}{\alpha_b - 1},$$

$$\alpha_b = \frac{r_b + r_g(\alpha_g - 1) + \alpha_g}{1 + r_b}, \quad \alpha_g = \frac{r_g + r_b(\alpha_b - 1) + \alpha_b}{1 + r_g}.$$

(and this happens if $\alpha_g = \alpha_b \to 1$, $\mathrm{r}_g = r_b \to 1$).

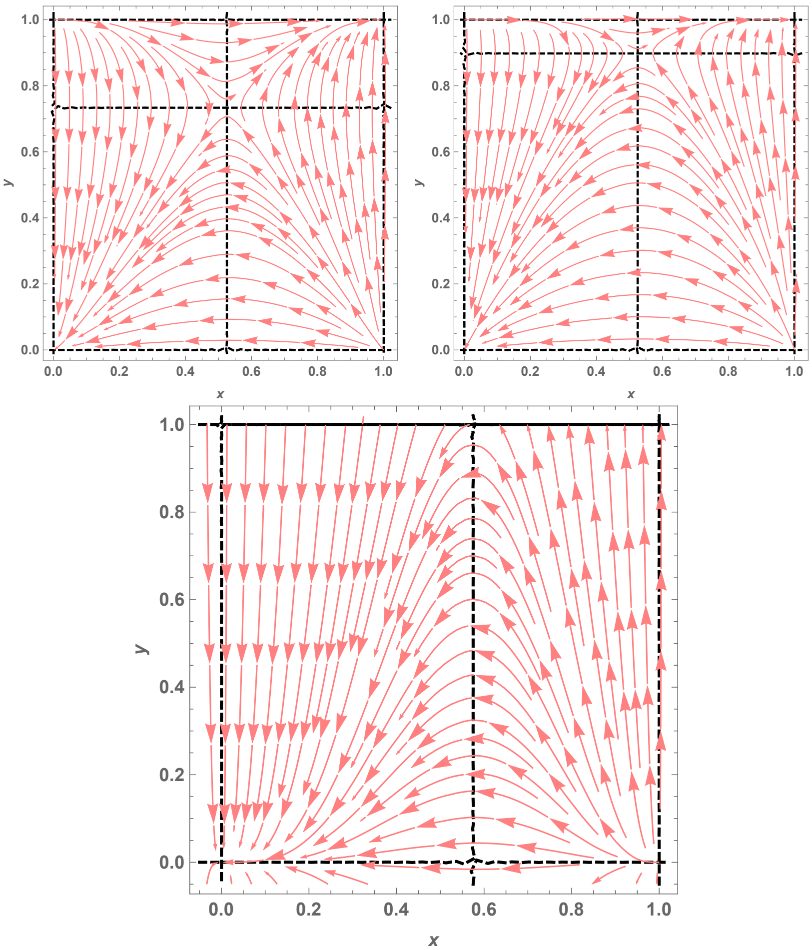

Figure 7: Phase portrait of the replicator system $(\dot{x}, \dot{y})$, where the interior equilibrium collapses at the upper edge of the unitary square. On such an upper edge arise a multiplicity of stable (right side) and unstable (left) fixed points.

Notice at the upper-edge of Figure 7 there is a wide segment of stable fixed points (multiple equilibria) that are given by: $(1-\bar{x}) = \frac{\pi_g - \pi_b + C(1+\tau - L\gamma_g)}{L(\gamma_b - \gamma_g)}$, while its counterpart is given by a set of unstable fixed points that will end at equilibrium $(0, 0)$ which is locally stable. In other words, this is a case of degenerate bifurcation such that if the economy overcomes the threshold value of green banks, $\bar{x}$, however it does not imply that the economy as a whole will end up in the green situation, since it will end up in one of the great variety of a multiplicity of equilibria with certain fractions of green banks (the rest is brown) while the firms are entirely green.

2. The interior equilibrium coalesces at the bottom-edge of the unitary square (Figure 8),

which means that under certain conditions, the economy as a whole (banks and firms) converge towards the green equilibrium once a certain threshold value is exceeded, but if such a threshold value is not exceeded, then the economy is characterized by a wide variety of stable fixed points (multiple equilibria) where all firms are brown and a fraction of banks is brown while the rest is green. That is, a bifurcation $((\bar{x}, \bar{y}) = (\bar{x}, 0))$ can occur if any of the following conditions are met:

$$r_g = \frac{r_b(\beta_b - 1) + \beta_b - \beta_g}{\beta_g - 1}, \quad r_b = \frac{r_g(\beta_g - 1) + \beta_g - \beta_b}{\beta_b - 1},$$

$$\beta_b = \frac{r_b + r_g(\beta_g - 1) + \beta_g}{1 + r_b}, \quad \beta_g = \frac{r_g + r_b(\beta_b - 1) + \beta_b}{1 + r_g}.$$

(and this happens if $\beta_g = \beta_b \to 1$, $r_g = r_b \to 1$)... Figure 8

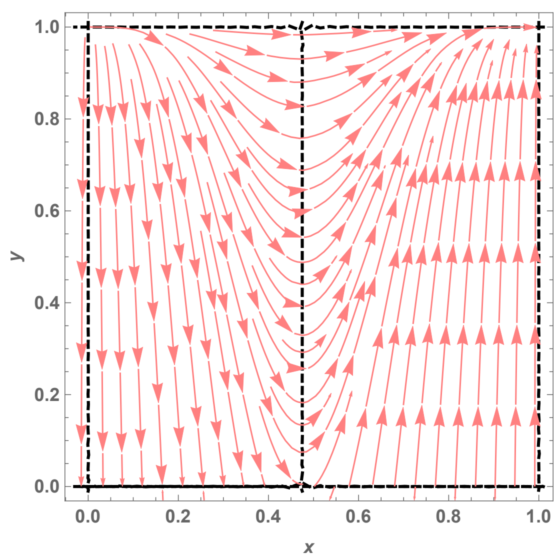

Figure 8: Phase portrait of the replicator system $(\dot{x}, \dot{y})$, where the interior equilibrium collapses at the bottom edge of the unitary square. On such a bottom edge arises a multiplicity of stable (left side) and unstable (right side) fixed points.

Totally contrary to what was stated in the previous item, notice that at the bottom-edge of Figure 8 there is a wide segment of unstable fixed points given by the value: $(1-\bar{x}) = \frac{\pi_g - \pi_b + C(1+\tau - L\gamma_g)}{L(\gamma_b - \gamma_g)}$, that will end up in equilibrium $(1, 1)$ which becomes locally stable. While its counterpart is given by a set of stable fixed points. In other words, this is a case of a degenerate bifurcation such that if the economy exceeds the threshold value of green banks, $\bar{x}$, it does imply that the economy as a whole will end up in the green situation. Otherwise it will end up in one of a great variety of a multiplicity of equilibria with certain fractions of green banks (the rest are brown) while the firms are entirely brown.

3. The interior equilibrium coalesces at the right-edge of the unitary square (see Figure 9), which means that all banks are green but there is a fraction of green and brown firms.That is, $((\bar{x}, \bar{y}) = (1, \bar{y}))$ which may occur if either:

$$\pi_g = \pi_b + L\gamma_g - C(1+\tau), \quad \pi_b = \pi_g - L\gamma_g + C(1+\tau), \quad L = \frac{\pi_g - \pi_b + C(1+\tau)}{\gamma_g},$$

$$\gamma_g = \frac{\pi_g - \pi_b + C(1+\tau)}{L}, \quad C = \frac{\pi_b - \pi_g + L\gamma_g}{1+\tau}, \quad \tau = \frac{\pi_b - \pi_g + L\gamma_g - C}{C}.$$

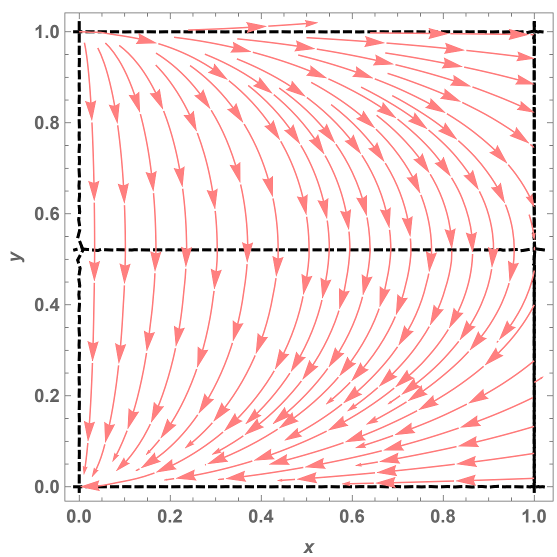

Figure 9: Phase portrait of the replicator system $(\dot{x}, \dot{y})$, where the interior equilibrium collapses at the right edge of the unitary square. On such a right edge arises a multiplicity of stable (upper side) and unstable (bottom side) fixed points.

Note that there is a wide variety of stable fixed points (multiple equilibria) if the threshold value $\bar{y}$ is exceeded, these multiple equilibria (fixed points) are given by the segment:

$$(1-\bar{y}) = \frac{r_g(1-\alpha_g) - r_b(1-\alpha_b) + \alpha_b - \alpha_g}{r_g(\beta_g+\alpha_g) + r_b(\alpha_b-\beta_b) + \alpha_b - \alpha_g + \beta_g - \beta_b}$$

while if such threshold value is not exceeded then the economy converges to the locally stable steady state of banks and brown firms $(0,0)$.

4. The interior equilibrium coalesces at the left-edge of the unitary square (see Figure 10), which means that all banks are brown but there are fractions of green and brown firms. That is, $((\bar{x},\bar{y}) = (0,\bar{y}))$ which can occur if any of the following equalities hold:

$$\pi_g = \pi_b + L\gamma_b - C(1+\tau), \quad \pi_b = \pi_g - L\gamma_b + C(1+\tau), \quad L = \frac{\pi_g - \pi_b + C(1+\tau)}{\gamma_b},$$

$$\gamma_b = \frac{\pi_g - \pi_b + C(1+\tau)}{L}, \quad C = \frac{\pi_b - \pi_g + L\gamma_b}{1+\tau}, \quad \tau = \frac{\pi_b - \pi_g + L\gamma_b - C}{C}.$$

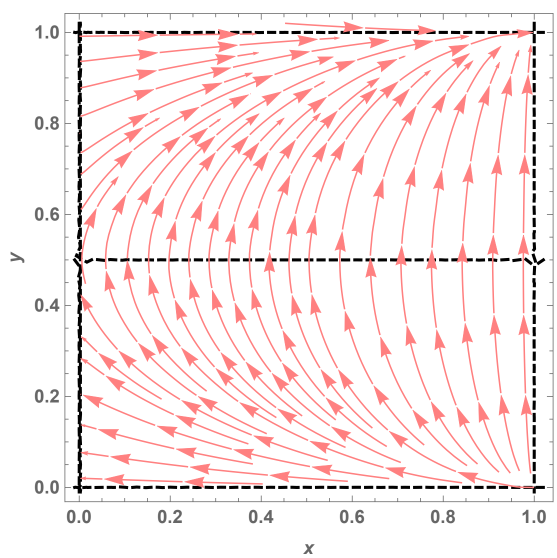

Figure 10: Phase portrait of the replicator system $(\dot{x}, \dot{y})$, where the interior equilibrium collapses at the left edge of the unitary square. On such a left edge arises a multiplicity of stable (bottom side) and unstable (upper side) fixed points.

Contrary to what was stated in the previous item, if the threshold value $\bar{y}$ is exceeded, then the economy converges to the situation of all banks and all firms are green $(1,1)$. Whereas if such a threshold value is not exceeded, then there is a large variety of stable fixed points (multiple equilibria) given by the segment:

$$(0 - \bar{y}) = \frac{r_g(1-\beta_g) - r_b(1-\beta_b) + \beta_b - \beta_g}{r_g(\beta_g - \alpha_g) + r_b(\alpha_b - \beta_b) + \alpha_b - \alpha_g + \beta_g - \beta_b}.$$

Let us now offer some final comments. In general terms, we can affirm that the revenues of green firms, the costs of the productive transition from brown to green, and green interest rates (as green monetary policy) considering the risks of non-performing loans, are the crucial parameters for an economy going or not towards a stable green system.

4. Concluding remarks

In this paper, we study the strategic interaction between greening or not from banks and firms in the economy. We propose a dynamic game in which firms that invest in green technologies face green transition costs that are proportional to the investment itself and loan costs that depend on the interest rate charged by banks. The latter might foster green technologies by lowering the interest rate on green loans, or not. From the bank point of view, lower interest rates render the loan less profitable than higher interest rates. Firms that decide to invest in standard technologies face production costs, a carbon tax, and loan costs. Moreover, the model considers firms' default probability, i.e., the loss banks face if firms do not repay their loans. Importantly, we assume that these probabilities depend on the type of firms (green or brown) and on the policy undertaken by the banks. Following previous economic findings according to which green firms are more resilient to adverse shock, the risk of not repaying the loan is the lowest for green firms that are experimenting with a green monetary policy, i.e., a green (and low) interest rate. We obtain several findings.

Firstly, five equilibria may occur:

- A brown-equilibrium in which the system do not move to a green economy.

- A green-equilibrium in which firms and banks are all green.

- Two mismatches, and an interior equilibrium in which brown and green firms coexist with brown and green banks.

Secondly, from the stability analyses, we find that when the equilibrium (B, B) is unstable and is a source, then the equilibrium (G, G) is the only asymptotically stable equilibrium, and the interior equilibrium point disappears. Similarly, when the equilibrium (G, G) is unstable (or nodal source), the equilibrium (B, B) is the only asymptotically stable one. The two corner mismatch equilibria (B, G) and (G, B) are always unstable.

Thirdly, there are cases of degenerate bifurcations, and in addition multiple equilibria can also arise. For example, when green interest rates or transition costs are high enough, then the segment of fixed points towards the brown level equilibrium increases. This is a key finding that suggests, on the one hand, that monetary policy is important for the green transition and, on the other, that innovation is also an important factor to the extent that it reduces such transition costs.

Fourthly, the transition to green might be realized regardless of green monetary policy, e.g., in the presence of a high carbon tax. This finding is in line with previous empirical evidence that underlines the carbon tax's importance in fostering environmentally friendly technologies.

Currently, there is a serious issue related to the tightening of monetary policies. Indeed, high interests rate increase the cost of capital thus discouraging or slowing the green transition because of the lower upfront costs of brown technologies. Moreover, such tightening may even expose economies to the risks of "climateflation" and "fossilflation" – that is, persistent inflationary pressures associated with more frequent natural disasters and a continued dependency on gas, oil, and coal.[12]

In this direction, even if our results indicate support for green monetary policies, we also

[12] Isabel Schnabel's Speech, Member of the Executive Board of the ECB, at the International Symposium on Central Bank Independence, Sveriges Riksbank, Stockhol: Monetary policy tightening and the green transition. *International Symposium on Central Bank Independence.* 10th January 2023, Stockholm. https://www.ecb.europa.eu/press/key/date/2023/html/ecb.sp230110~21c89bef1b.en.html

show that decarbonization might occur regardless of low-interest rates, in the measure that the carbon tax is high enough and (or) the cost of green transition is low enough. Indeed, there are several market imperfections and resource constraints that limit the investment in green technologies (Bacchiocchi *et al.* (2024b)) from shortages of funds, capabilities and skills, to inefficient infrastructures and technologies and low competition (OECD (2019)).

Our future research agenda aims at augmenting the analysis in three directions. First, we aim to account for changes in consumer attitudes towards green firms. Secondly, default probabilities are assumed given, but we could model them as inversely related to the revenues of the firms (for instance: Bellucci *et al.* (2023), Calcagnini *et al.* (2018)). Finally, we aim to study the interaction effects of fiscal, monetary, and innovation policies on green transition.

Conflict of Interest Statement: The authors declare that they have no conflict of interest.

Appendix A. Stability conditions

We can assess whether the five equilibria are *asymptotically stable points* by analyzing the *Jacobian matrix* $J(\dot{x}, \dot{y})$, such that equilibria fitting $\det(J) > 0$ and $tr(J) < 0$ are asymptotically stable. That is, the local stability of the each equilibrium point is determined through the usual linearization procedure, i.e., according to the study of the eigenvalues of the Jacobian matrix,[13]

$$J(x,y) = \begin{bmatrix} \begin{array}{c} L(-1+2x)\left[y(\alpha_g-\alpha_b+\beta_b-\beta_g)+r_g(y(\alpha_g-\beta_g)+\beta_g-1)-\right. \\ \left.-r_b(y(\alpha_b-\beta_b)+\beta_b-1)+\beta_g-\beta_b\right. \end{array} & \begin{array}{c} L(-1+x)x\left[\alpha_g(1+r_g)-\alpha_b(1+r_b)+\right. \\ \left.+\beta_b(1+r_b)-\beta_g(1+r_g)\right) \end{array} \\ L(1-y)y\left(\gamma_b-\gamma_g\right) & (1-2y)\left[\pi_g-\pi-b+C(1+\tau)-L(1-x)\gamma_b-Lx\gamma_g\right]] \end{bmatrix}$$

computed at each of the the equilibrium points. Let us first analyze this Jacobian matrix in the interior equilibrium $(\bar{x}, \bar{y})$, i.e.

$$J(\bar{x},\bar{y}) = \begin{bmatrix} 0 & J_{12} \\ J_{21} & 0 \end{bmatrix}$$

where

$$J_{12} = -\frac{\left[\pi_g-\pi_b+C(1+\tau)-L\gamma_g\right]\left[(\pi_g-\pi_b+C(1+\tau)-L\gamma_b)((\beta_g-\alpha_g)(1+r_g)+(\alpha_g+\beta_g)(1+r_b))\right]}{L(\gamma_b-\gamma_g)^2},$$

and

$$J_{21} = -\frac{L\left[\alpha_b(1+r_b)-\alpha_g(1+r_g)+r_g-r_b\right]\left[\beta_b(1+r_b)-\beta_g(1+r_g)+r_g-r_b\right](\gamma_b-\gamma_g)}{\left(\beta_g(1+r_g)-\alpha_g(1+r_g)+(1+r_b)(\alpha_b-\beta_b)\right)^2}.$$

The sufficient conditions for the stability of an equilibrium point, i.e. the conditions for both the eigenvalues have negative real part, are given by the Routh-Hurwitz Criterion (Bischi

[13] The idea is to use the eigenvalues of the Jacobian matrix evaluated at a critical point to understand the behavior of the system near that critical point (Bischi *et al.* (2016).

et al. (2016)), that is the trace, $Tr(J) < 0$, and the determinant, $Det(J) > 0$, of the Jacobian matrix. The first condition is never satisfied when the equilibrium $(\bar{x}, \bar{y})$ exists inside the unit square, since $Tr(J) = 0$. Hence such equilibrium maybe a saddle point (see Figure 4) if $Det(J) < 0$, i.e. $-(J_{12} \times J_{21}) < 0$, which holds if:

$$\pi_g - L\gamma_g > \pi_b - C(1+\tau), \quad \pi_g - L\gamma_b > \pi_b - C(1+\tau),$$

$$r_b(\alpha_b - 1) > r_g(\alpha_g - 1) + \alpha_g - \alpha_b, \quad r_b(\beta_b - 1) > r_g(\beta_g - 1) + \beta_g - \beta_b.$$

Whereas $(\bar{x}, \bar{y})$ is a center (see Figure 5) if $Det(J) = 0$, i.e. $-(J_{12} \times J_{21}) = 0$ and this happens when the previous conditions of strict inequality become conditions with an equality or equivalence sign, that is:

$$\pi_g = \pi_b + L\gamma_b - C(1+\tau), \;\; \pi_g = \pi_b + L\gamma_g - C(1+\tau),$$

$$r_g = \frac{r_b(\alpha_b - 1) + \alpha_b - \alpha_g}{\alpha_g - 1}, r_g = \frac{r_b(\beta_b - 1) + \beta_b - \beta_g}{\beta_g - 1}.$$

All this at least according to linearization, or equivalently, the eigenvalues of $J(\bar{x}, \bar{y})$ are readily found to be $\lambda = \pm i$, which confirms that this equilibrium is a center for the linearized system.

The stability analysis at the corner equilibria is straightforward, as the Jacobian matrix is diagonal at any corner equilibrium, hence the eigenvalues are given by the diagonal entries. For example, at $(0, 0)$ we have

$$J(0,0) = \begin{bmatrix} L[r_g(1-\beta_g) - r_b(1-\beta_b) + \beta_b - \beta_g] & 0 \\ 0 & \pi_g - \pi_b + C(1+\tau) - L\gamma_b \end{bmatrix}$$

hence, being the eigenvalues $\pi_g - \pi_b + C(1+\tau) - L\gamma_b < 0$ and $L[r_g(1-\beta_g) - r_b(1-\beta_b) + \beta_b - \beta_g] < 0$, this implies that this equilibrium $(0, 0)$ is a stable node, whereas it is a unstable if the inequalities are reversed. Instead it can be a saddle if $\pi_g - \pi_b + C(1+\tau) - L\gamma_b < 0$ and

$L[r_g - r_b(1-\beta_b) + \beta_b - \beta_g(1+r_g)] > 0$ with unstable manifold along the vertical axis.

Analogously, for the equilibrium point $(1,1)$,

$$J(1,1) = \begin{bmatrix} L[\alpha_g - \alpha_b + r_b(1-\alpha_b) - r_g(1-\alpha_g)] & 0 \\ 0 & \pi_b - \pi_g - C(1+\tau) + L\gamma_g \end{bmatrix}$$

from the conditions of negativity of the two diagonal entries we get the conditions for $(1,1)$ being a stable node, whereas if both the inequalities are reversed it is an unstable node and if only one is reversed, i.e. the eigenvalues are one negative and one positive, then $(1,1)$ is a saddle point with stable and unstable manifolds along the horizontal and vertical directions.

Analogous arguments can be applied to the other two corner equilibria. That is, for the equilibrium point $(0,1)$

$$J(0,1) = \begin{bmatrix} L[\alpha_b - \alpha_g + r_g(1-\alpha_g) + r_b(1-\alpha_b)] & 0 \\ 0 & \pi_b - \pi_g - C(1+\tau) + L\gamma_b \end{bmatrix}$$

and the equilibrium point $(1,0)$,

$$J(1,0) = \begin{bmatrix} L[\beta_g - \beta_b - r_g(1-\beta_g) + r_b(1-\beta_b)] & 0 \\ 0 & \pi_g - \pi_b + C(1+\tau) - L\gamma_g \end{bmatrix}$$

and so they are unstable points (see the above Figures 2-3) since $r_g(1-\alpha_g) > r_b(1-\alpha_b)$ and $\pi_g + L\gamma_g > \pi_b + C(1+\tau)$.

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