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Via delle Pandette 9, 50127 Firenze (Italia) www.disei.unifi.it

Economic Growth, Poverty Traps, and Wealth Concentration: Riddles and Waves Driven by Unproductive Assets

Francesca Grassetti[a,*], Edgar J. Sanchez Carrera[b]

[a] *Department of Economics, Society, and Politics, University of Urbino Carlo Bo, Italy. E-mail: francesca.grassetti@uniurb.it*

[b] *Department of Economics and Management (DISEI), University of Florence, Italy. Research Fellow at the Applied Mathematics Research Center CIMA UAdeC, Mexico. E-mail: edgar.sanchez@unifi.it ORCID: 0000-0002-1191-8859*

Abstract

This research paper studies a non-linear dynamic macroeconomic model to analyze the interactions between economic growth and the concentration of wealth driven by productive and unproductive capital or bubbly assets. The paper considers both a deterministic and a stochastic version of the model to analyze the economy's evolution, i.e., growth, poverty traps, and wealth concentration. For the deterministic case of the model, we show that poverty traps exist as a stable or unstable (saddle) steady state, depending on whether the returns to unproductive assets are positive or negative. A high-level equilibrium exists when the returns on unproductive assets are negative, but fluctuations (i.e., cycles or oscillations) may occur. For the stochastic and bubbly case, we calibrate the model, and we may conclude that the implementation of economic policies that improve the investments of capitalists in productive capital would remove the economies from the threat of a poverty trap and possible fluctuations, in addition to the fact that wealth inequality would decline in the long run.

Keywords: Economic Growth; Non-linear Growth Models; Rentier premium; Unproductive assets; Wealth inequality.

JEL classification: E10, E21, O16, O40, P46

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[*] Corresponding author. Email: francesca.grassetti@uniurb.it

1. Introduction

This paper contributes to the extensive literature on wealth (income) inequality, economic growth, and poverty traps (since, for example, the seminal contributions of Azariadis, 1996; Galor and Zeira, 1993; Ghiglino and Sorger, 2002). In particular, we study a key aspect that has not been well analyzed or taken into account. Namely, wealth inequality and divergences in economic growth across countries are due to capital investment in unproductive activities (bubble financial or rent-seeking activities).

Since financial professionals consider some financial assets to be disconnected from the real economy, their sole purpose is to shift rents away from one sector of the economy (i.e., the productive one) in favor of the rentier class. The distinction between productive capital K and wealth W, the latter including these speculative financial assets and durable luxury goods, is crucial to explaining how wealth accumulation can further increase inequality and slow economic growth. Piketty (2014) has drawn the scientific community's attention to this distinction between types of capital. This has led Stiglitz (2016) to note that:

> ”Standard data on savings cannot be reconciled with the increase in the wealth-income inequality ratio: there is a wealth residual. An important component of this is associated with rents: land rents, exploitation rents, and rents on intellectual property.”

What can be stated is that wealth inequality goes strictly hand in hand with financial activities (Levine, 2021; Botta et al., 2021) or with the acquisition of unproductive assets, that is, the accumulation of financial assets such as housing, land, luxury goods, deposits, bonds, financial shares (Holmes and Jr, 2001; Policardo and Sanchez Carrera, 2024).[1] The

[1]Even if the wealth-income ratio increases, the wealthiest people will stop investing in productive capital, as the additional capital units become almost worthless (Piketty and Saez, 2014; Poterba, 2000; Stiglitz, 2016). Piketty (2014) asserts that capital assets may assume all forms of real property (including residential estate), financial capital, and functional capital like plant, infrastructure, machinery, patents, and so on, used by firms and governmental agencies. Capital intended as a means of production is only a subset of this broader definition of capital. Certain luxury consumer durables, like art, boats, wine collections, and fancy

standard way unproductive assets are dealt with in economics is through bubbles, assets that pay zero dividends but may bring in capital gains. Major examples are discussed in the seminal contributions by Samuelson (1958), Tirole (1985), Grossman and Yanagawa (1993) and Martin and Ventura (2012). In this vein, our research paper follows the work of Martin and Ventura (2012), who pointed out that in an economy with two different types of agents, one productive and the other unproductive, bubbles transfer resources between these agents, thus mitigating the effects of financial frictions. However, such resource bubble transfers can deplete the economy's capital stock and output. These results are interesting, but one limitation is that they are based on a neoclassical growth framework. Hence Ho and Zhu (2019) extends the framework of Martin and Ventura (2012) to an environment in which growth is driven by capital production (Romer (1986)), and showing that the degree of unproductive agents' productivity is critical to the usefulness of bubbles even when the quality of the financial system is poor (i.e., the degree of pledgeability is very low).[2]

It could be argued, then, that the theory of economic growth is governed by the theory of consumption or by the theory of investment decisions in productive and unproductive business activities (some well-known contributions are due to: Duesenberry, 1949; Baudet and van der Meulen, 1982; Poterba, 2000; Holmes and Jr, 2001; Amendola et al., 2017; Botta et al., 2021), stating that as income, and therefore wealth, increases, then entrepreneurs, individuals or families disproportionately purchase and invest in more unproductive assets (on this, see the contributions of Amendola et al., 2017; Browning and Crossley, 2014; Claudio, 2018; Holmes and Jr, 2001; Tsoulfidis and Paitaridis, 2019; among others). In other words, as personal and household wealth increases, people spend an increasing percentage

cars, are particularly attractive to the rich as stores of value and status indicators but are not productive capital.

[2]Analyzing the set of transfers that bubbles implement is useful for understanding the effects of bubbling episodes. Remember that a bubble is nothing more than a pyramid scheme whereby the buyer provides resources today, hoping that future buyers will give them. The economy enters each period with an initial distribution of bubble owners. Therefore, a key aspect of the theory is how the distribution of bubble owners is determined. At the same time, the general confusion occurs between stocks and flows. The income stream is the return on the stock of wealth. But the returns to wealth vary dramatically from person to person and country, so it is necessary to distinguish between income and wealth (Hamilton and Hepburn, 2014).

of their income on non-productive activities, and the proportion of spending on these assets continues to grow as wealth increases. That is, economic agents consume and invest more in, for example, luxury goods or unproductive assets, and it is thus that the concentration of wealth driven by the consumption and investment of these types of assets permeates the performance of an economy. In this vein, Claudio (2018) shows that poor agents who consume a lower proportion of luxury goods are more risk averse than rich agents, so they have stronger precautionary motives, forcing towards a more equitable distribution of wealth. The rich are known to save more of their income than the poor, which will naturally increase wealth inequality relative to income inequality (see, Alvaredo et al. (2017)). However, as Stiglitz (2016) notes, this difference in savings rates is insufficient to explain wealth inequality growth in recent decades. What Stiglitz identifies as rents, which we broadly consider asset appreciation, is also at play.

All of the literature above is the motivation for this research paper. However, unlike this literature, we offer a new explanation for why poverty traps arise and why there are riddles and waves in economic growth. The reason is that the economy is driven by investment in unproductive assets.[3] To reach our aim, we study an economy with two types of agents: capitalists and workers. The former produces capital, and the latter provides a labor force. Part of the capitalists' capital income is invested to become unproductive wealth, which generates certain returns. Another part of capitalists' capital income is saved and put into productive assets (i.e., capital stock), generating returns. A representative capitalist's total wealth is the sum of returns from the two (productive and unproductive) assets. Similarly, a proportion of workers' wage income in each period is saved to become unproductive wealth,

[3]Since the evidence shows us that the concentration of wealth is no longer coming from productive activities, it may be from rent-seeking activities or even connections with people in power, that is, political connections. As Bagchi and Svejnar (2015) show, wealthy people who acquire their wealth through political connections, so there is a strict relationship between political connections and wealth accumulation, may negatively impact economic growth. In contrast, the opposite is not significant at all. As suggested by Lee (2021) and based on the empirical backgrounds and empirical evidence of heterogeneous returns to wealth (Fagereng et al., 2016; Bach et al., 2020; Cao and Luo, 2017), the model we propose presents the rich as a 'rentier class', defined as those who have a monopoly of access to a specific type of asset (physical, financial, intellectual, etc.) which is an unproductive asset for the economic production process.

and the worker's wage income in the current period is equal to his productive wealth. A representative worker's total wealth is the sum of returns from these two assets. This economy has a CES production function with capital stock and labor force as inputs. In a deterministic version of this framework, we argue that poverty traps are possible to emerge depending on three major factors, including the saving rate of the capitalists on productive assets, the rate of returns to unproductive assets, and the degree of substitution between capital stock and labor in the CES production function. In a stochastic version of this framework, we find that changing capitalists' saving rates can help the economy avoid poverty traps and economic fluctuations.[4]

The remainder of this paper is structured as follows to achieve our goals. Section 2 develops the dynamic model of the economy to be analyzed. We obtain a dynamic mapping for such an economic representation. Section 3 gives the results of such a dynamic economy, according to the cases of the rate of return of unproductive assets, thus obtaining poverty traps, fluctuations, and economic inequalities. Section 4 concludes and gives hints for future research.

2. The economy

We represent an economy of competitive profit-maximizing firms, which capitalists and workers run. Either capitalists and workers may distribute their wealth between productive capital (an exclusive action for capitalists) and unproductive capital (an action that either capitalists or workers can take). Then, the capitalist's and workers' wealth is accumulated through remunerations and returns on productive and unproductive capital assets. At the same time, the economy grows according to a law of motion of productive capital accumulation. To exemplify and schematize the economy, we present Figure 1, which summarizes the model's timeline.

[4]Basically we consider that the return on unproductive (bubble) capital, $r_{u,t}$, is measured as a function of the return on productive capital, $r_{k,t}$, where the former may have a higher or lower return with respect to productive capital but also a negative return on that capital, or ultimately it may have a stochastic return.

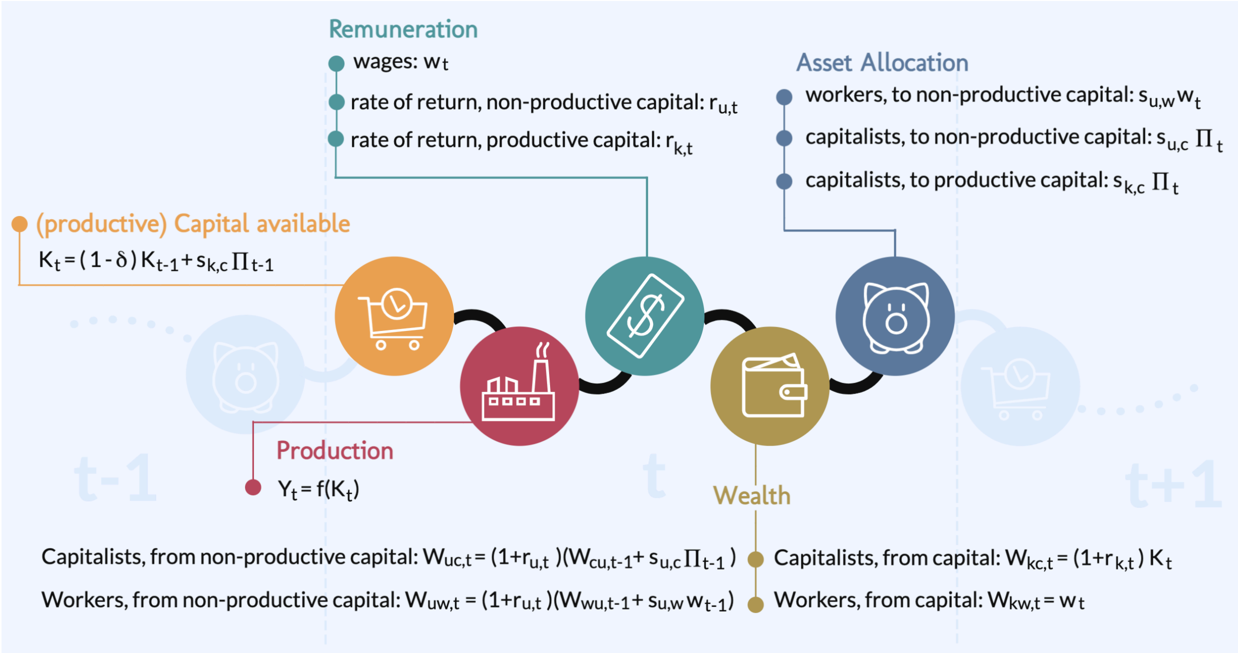

Figure 1: Timeline of the model.

Mainly, time is discrete and at each moment $t \in \mathbb{N}^{+}$, the competitive firm that maximizes its profits has a technology defined by the ACMS production function,[5]

$$Y_t = f(K_t) = \gamma[\beta K_t^{-\rho} + (1-\beta)L_t^{-\rho}]^{-1/\rho} \tag{1}$$

where $Y > 0$ is the produced output, $K > 0$ denotes the physical capital stock, $L > 0$ is the labor input, $\gamma > 0$ is the neutral efficiency parameter (it changes the output for every given set of inputs in the same proportion) and $\beta \in (0,1)$ is the distribution parameter (it modifies the functional distribution of income). The substitution parameter ρ determines the well-known elasticity of substitution $\sigma = 1/1+\rho$, i.e., the ratio of percentage change in capital-labor ratio with the percentage change in Marginal Rate of Technical Substitution. As discussed in Arrow et al. (1961), the permissible values for ρ run from -1 to $+\infty$, but

[5]In 1961, Kenneth Arrow, H.B. Chenery, B.S. Minhas and Robert Solow developed what became known as the Arrow-Chenery-Minhas-Solow or ACMS production function Arrow et al. (1961). Later in the literature, this became known as the constant elasticity of substitution or CES production function. This function allowed the elasticity of substitution to vary between zero and infinity. Once this value was established, it would remain constant across all output and/or input levels.

empirical values for σ are usually significantly less than one. Accordingly, we assume $\rho > 0$ which leads to $\sigma < 1$.

The economy is populated by two infinite-life agents: *capitalists*, c, who earn income from capital, and *workers*, w, who receive income-wage from labor. Capitalists run firms and thus maximize profits subject to (1). Therefore, wages are equal to the marginal productivity of labor:

$$w_t = \frac{\partial f(K_t)}{\partial L_t} L_t = \gamma(1-\beta)L_t^{-\rho}[\beta K_t^{-\rho} + (1-\beta)L_t^{-\rho}]^{-\frac{1+\rho}{\rho}}, \tag{2}$$

while the marginal product of capital is defined by the real interest rate (Giammarioli and Annicchiarico (2004)):

$$r_{k,t} = \frac{\partial f(K_t)}{\partial K_t} = \gamma\beta[\beta K_t^{-\rho} + (1-\beta)L_t^{-\rho}]^{-\frac{1+\rho}{\rho}} K_t^{-(1+\rho)}, \tag{3}$$

and, consequently, the profits are given by:

$$\Pi_t = \frac{\partial f(K_t)}{\partial K_t} K_t = r_{k,t} K_t. \tag{4}$$

As pointed out by Stiglitz (2015) the recent structural changes in income distribution are mainly due to rent-seeking:

> Rent-seeking means getting an income not as a reward for creating wealth but by grabbing a larger share of the wealth that would have been produced anyway.

While for neoclassical economic theories, the term wealth refers to "capital", recent academic research such as Stiglitz (2015) observes that rents are increasing due to land and property rents, intellectual property rents, and monopoly power. Thus, it also increases the value of those non-productive assets that can provide income to their owners. It follows that overall wealth increases, but since the assets that drive the rise in wealth do not produce capital goods (and are not 'productive' in the ordinary sense), this increase does not lead to the greater productive capacity of the economy. In other words, wealth increases due to the accumulation of unproductive assets, and this phenomenon affects the marginal productivity of the productive factors and, consequently, the distribution of income. Since productive capital has a rate of return equal to the interest rate, there is no clearly defined rate of return for unproductive capital goods (i.e., rental activities, possession of luxury goods, etc.). In what follows, we propose formalizing these considerations, assuming two investment opportunities exist. That is, agents can distribute their wealth between productive capital/asset paid at a rate of return $r_{k,t}$, and an unproductive-bubble asset in the sense of Stiglitz (2015) with a rate of return defined as:

$$r_{u,t} = \phi r_{k,t} + \varepsilon_t \tag{5}$$

where $\phi > -1$ is the positive or negative return on unproductive capital measured in terms of the return on productive capital. The term $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$ is a white noise with variance σ_ε^2; this is the stochastic part of the return on unproductive capital. In other words, the return on unproductive capital, $r_{u,t}$, is measured as a function of the return on productive capital, $r_{k,t}$, where the former may have a higher return with respect to the productive capital but also a negative return with respect to such capital, or ultimately it may have a stochastic return. It should be noted that it may be puzzling that the return on unproductive capital can take on a negative value (i.e., $\phi < 0$) since, given this situation, a rational agent would likely have no incentive to hold any amount of unproductive assets. However, this should not be misleading, as the return on unproductive assets takes on a negative value relative to the return on productive capital and has a stochastic component that complicates the agent's decision about whether or not to hold unproductive assets.

Let us denote the proportions of income allocated to unproductive capital by $s_{u,c}$ and $s_{u,w}$ for capitalists and workers, respectively. In addition, capitalists invest a constant part (saving rate) $s_{k,c}$ of their profits in productive capital. Thus, $s_{k,c} \in [0, 1]$, $s_{u,w} \in [0, 1]$ and $s_{u,c} \in [0,\ 1 - s_{k,c}]$. We highlight that although in current growth models, the savings rate is usually defined as the output of an intertemporal decision of the agent, in this initial work, we prefer to take the saving decision as given in neoclassical theory (see Cersatto (1999)). In the case of endogenously generated savings rates, we defer to future developments in the long-run analysis. In each period of time, the capitalist's wealth accumulation before investing ($W_{c,t}$) is made up of their income generated from productive capital,

$$W_{kc,t} = (1 + r_{k,t})K_t \tag{6}$$

and that generated by unproductive capital,

$$W_{uc,t} = (1 + r_{u,t})(W_{uc,t-1} + s_{u,c}\Pi_{t-1}), \quad W_{uc,0} = 0, \tag{7}$$

so total wealth of capitalists is:

$$W_{c,t} = W_{kc,t} + W_{uc,t}.$$

Instead, the wealth of workers is resulting from productive capital through the labor productivity that follows in wages (the above equation (2)),

$$W_{kw,t} = w_t \tag{8}$$

and from investments in unproductive capital,

$$W_{uw,t} = (1 + r_{u,t})(W_{uw,t-1} + s_{u,w}w_{t-1}), \quad W_{uw,0} = 0. \tag{9}$$

So the total wealth of workers is:

$$W_{w,t} = W_{kw,t} + W_{uw,t}.$$

While further research may investigate the case in which the labor force evolves,[6] the effect

[6]Without loss of generality and simplicity, let us assume that labor input is constant and normalized to one, i.e., $L_t = L = 1$.

of asset allocation driven by optimization choices, and the case of $r_{u,t}$ linked to the demand of the unproductive asset or bubble. That is, to endogenize consumption/saving/portfolio decisions and explain why capitalists favor bubbles when their wealth increases. The Blanchard/Yaari model (Yaari (1965) and Blanchard (1985); Benhabib et al. (2016)) is a good place to start, particularly if we assume that workers do not trade in financial markets. The Blanchard/Yaari model with neoclassical technology, logarithmic utility with discount factor $\beta < 1$, perfect certainty, homogeneous infinitely lived cohorts, and constant population growth will deliver constant saving rates if: i) workers are autarkic hand-to-mouth households, and ii) capitalists do not supply labor. In this case, capitalists will save a fraction β of their capital wealth in all circumstances if utility is logarithmic, or a share of wealth that depends on market yield alone, if utility is isoelastic.[7]

Furthermore, the economy grows with a greater accumulation of productive capital; for this reason we consider that such productive capital accumulates and evolves according to the:

$$K_t = (1-\delta)K_{t-1} + s_{k,c}\Pi_{t-1} = (1-\delta + s_{k,c}r_{k,t-1})K_{t-1} \tag{10}$$

where $\delta \in (0,1)$ is the depreciation rate of capital and $s_{k,c}\Pi_t$ is the share of profits allocated to productive capital.

So given the above assumptions and substituting equation (3) into (10), equation (4) into (7), and (2) in (9), the dynamic evolution of the economy is defined by the following map (a non-linear dynamic system):

$$T = \begin{cases} K_{t+1} = & (1-\delta)K_t + s_{k,c}\gamma\beta\left[1+\beta\left(K_t^{-\rho}-1\right)\right]^{-\frac{1+\rho}{\rho}} K_t^{-\rho} \\ W_{uc,t+1} = & (1+r_{u,t+1})\left\{W_{uc,t} + s_{u,c}\gamma\beta\left[1+\beta\left(K_t^{-\rho}-1\right)\right]^{-\frac{1+\rho}{\rho}} K_t^{-\rho}\right\} \\ W_{uw,t+1} = & (1+r_{u,t+1})\left\{W_{uw,t} + s_{u,w}\gamma(1-\beta)[1+\beta(K_t^{-\rho}-1)]^{-\frac{1+\rho}{\rho}}\right\} \end{cases} \tag{11}$$

[7] This sounds fine, but equilibrium sequences of capital will be monotone, meaning there will be no business cycles. For that, we will need the unproductive asset or bubble.

subject to

$$\begin{array}{ll} r_{u,t} = \phi r_{k,t} + \varepsilon_t & \varepsilon_t \sim WN(0, \sigma_\varepsilon^2) \\ W_{uc,0} = 0 & W_{uw,0} = 0 \end{array},$$

Note that we must consider the economic variables related to the total wealth of workers and capitalists, that is:

$$\begin{aligned} W_{kc,t+1} &= (1 + r_{k,t+1})K_{t+1} \\ W_{kw,t+1} &= \gamma(1-\beta)[1+\beta(K_t^{-\rho} - 1)]^{-\frac{1+\rho}{\rho}} \\ W_{c,t+1} &= W_{kc,t+1} + W_{uc,t+1} \\ W_{w,t+1} &= W_{kw,t+1} + W_{uw,t+1} \end{aligned}$$

In the next section, we present the solutions for this economy. We show that there are mainly three cases, namely: i) an evolution of the economy towards poverty traps (proper and improper), ii) an evolution towards a stable positive equilibrium that, under certain conditions, loses its stability, thus characterizing the fluctuations (cycles or oscillations) economic, and iii) the stochastic and bubble volatility of the economy that corroborates how the concentration of wealth may decline.

3. Results of the model

Let us first discuss the long-term evolution of the economy considering the deterministic skeleton of the map (11), i.e., $\sigma_\varepsilon^2 = 0$, to understand how the endogenous components of the model influence its long-term evolution. Proofs of all results are in Appendix (Appendix A). We then calibrate the model to analyze the stochastic and bubbly case; although the stochastic volatility of non-earning assets in (5) is a key ingredient of the model, we can shed some light by stating that investment in productive activities is the critical policy for sustained economic growth and reducing the concentration of wealth.

3.1. Evolution towards poverty traps - deterministic skeleton of the model

The equilibrium of poverty traps exists in this economic model that we are developing; the poverty trap is an equilibrium defined by $E = (K, W_{uc}, W_{uw}) = (0, 0, 0)$, which is a low-level equilibrium such that economies do not grow because the productive capital accumulation is zero, and also to the fact that the wealth of their agents, both capitalists and workers, is zero.

Given that $\phi > 0$ is the most common case in which unproductive capital provides positive returns, the poverty trap presents two different types of instability: i) the poverty trap is such that the wealth resulting from investment in unproductive capital will increase due to the effect of ϕ, while the share of wealth coming from productive capital will converge to the poverty trap (in this case the fixed point is a saddle) if the saving rate of capitalists $s_{k,c}$ is low $\left(s_{k,c} \leq \delta\beta^{\frac{1}{\rho}}\gamma^{-1}\right)$, and ii) otherwise the economy diverges, which means that in this case the fixed point is unstable. This last scenario has no other fixed points (positive returns on unproductive assets). It is essential to analyze the global evolution of the model. Hence, we characterize two different types of poverty traps:

1. A *proper poverty trap* such that all accumulation of capital and wealth is zero. Such a poverty trap is a stable fixed point

2. An *improper poverty trap* such that the accumulation of capital and wealth generated by productive assets is zero, but the accumulation of unproductive assets grows. Such a poverty trap is a saddle-fixed point.

It is desirable that the fixed point E, called the poverty trap, be unstable. In fact, in the case of a saddle, this would determine that at least the portion of wealth derived from productive capital (usually in the hands of workers for the most part) would converge to the poverty trap in the long run and call it an "improper poverty trap."

The qualitative behavior of the system is affected by the parameter ρ, which determines the marginal rate of technical substitution or capital-labor ratio, $\sigma = \frac{1}{1+\rho}$. As Mallick (2012) demonstrated, the values of σ characterize different scenarios an economy may encounter during its development process. Low values of σ characterize undeveloped economies, while developed countries register higher values. The highest values of σ are generally experienced by those countries with the highest growth rates, that is, developing (emerging) countries. According to the results in Mallick (2012), we consider that:

1. when $\sigma \leq 0.2$ so we represent underdeveloped countries
2. when $0.2 < \sigma \leq 0.8$ so we represent developed countries, and
3. when $\sigma \geq 0.8$ then we represent developing (emerging) countries.

Let us analyze the change in qualitative behavior by moving the values of $s_{k,c}$ and σ.[8] As visible in Figure 2, the three types of economies (underdeveloped, developing, and developed) are all threatened by the improper poverty trap (case where E is a saddle), and the discriminating factor is the rate of investment in productive capital by capitalists $s_{k,c}$.

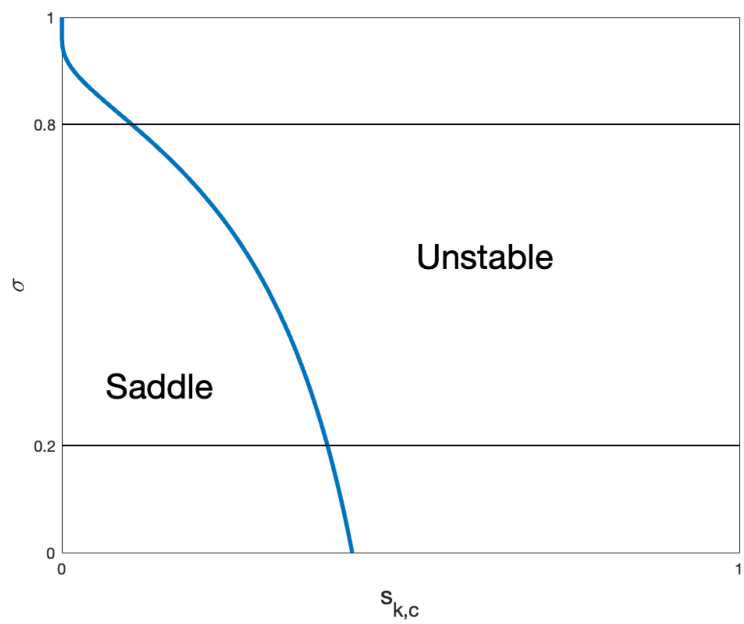

Figure 2: Bifurcation curve (in blue) and stability of the fixed point E moving $s_{k,c}$ and σ. Parameter values: $\delta = 0.3, \beta = \gamma = 0.7$.

[8]Obviously, the initial value of k (capital-labor ratio) which is used to determine the development level of a country is closely related to the initial value of capitalists' saving rate, $s_{k,c}$.

Notice that in Figure 2 the low levels of investment determine the transition between an unstable fixed point (and therefore non-convergence to the poverty trap) and a saddle. The lower the value of σ, the higher the number of points in the parameter plane $(s_{k,c}, \sigma)$ corresponding to the improper poverty trap. These observations lead us to the following statement.

Lemma 1. *When the return* (ϕ) *on unproductive capital is positive, economies are threatened by an improper poverty trap (saddle) in which productive capital goes to zero if capitalists invest too little in productive capital. Undeveloped economies need more investment in productive capital (higher $s_{k,c}$) to escape such a poverty trap. At the same time, developing countries are less prone to stagnation, even for low levels of $s_{k,c}$.*

For $\phi > 0$, while the poverty trap may be a saddle or an unstable fixed point, no other equilibria exist, and it is necessary to investigate the global dynamics of the total wealth of workers and capitalists. Although economic growth and wealth accumulation derived from productive capital can be in a poverty trap (due to the low values of the saving rate), the overall wealth of both workers and capitalists grows over time due to the effect of the interest rate concerning unproductive capital. It is important to analyze if such growth in wealth is uniform for both groups of agents or if there is a concentration of wealth. To do this, we study the difference in well-being or wealth accumulation in the long run between the two groups, considering different values of the savings rate and the elasticity of substitution. Then, we can state the following propositions.

Proposition 1. *For $\phi > 0$, developing countries have the lowest level of inequality between workers and capitalists, while developed countries, and even more so, underdeveloped countries, experience the highest levels of wealth inequality. Unfortunately, inequality increases in developed and underdeveloped economies when capitalists invest heavily in productive capital. That is, it is due to the allocation of resources to unproductive capital assets.*

Proof. In Figure 3, each point represents the ratio of the difference in wealth between workers and capitalists (this is wealth inequality), calculated as $(W_{c,t} - W_{w,t})/W_{w,t}$, for a given combination of $s_{k,c}$ and σ. The analysis is done after a thousand iterations of the model (t =1000) to show what happens in the long term. The Figure 3 of the level of inequality shows that:

- When capitalists allocate almost all the resources to unproductive capital ($s_{k,c}$ low), the economies that experience the highest values of inequality are underdeveloped countries, followed by developed countries.
- Moreover, regardless of the type of country under consideration, the inequality ratio is increasing in σ and $s_{k,c}$. The qualitative result still holds for other combinations of parameter values (such as $\phi > 1$.)
- Therefore, developing countries are the least likely to experience high levels of inequality, while developed and underdeveloped countries are where inequality is most pronounced.

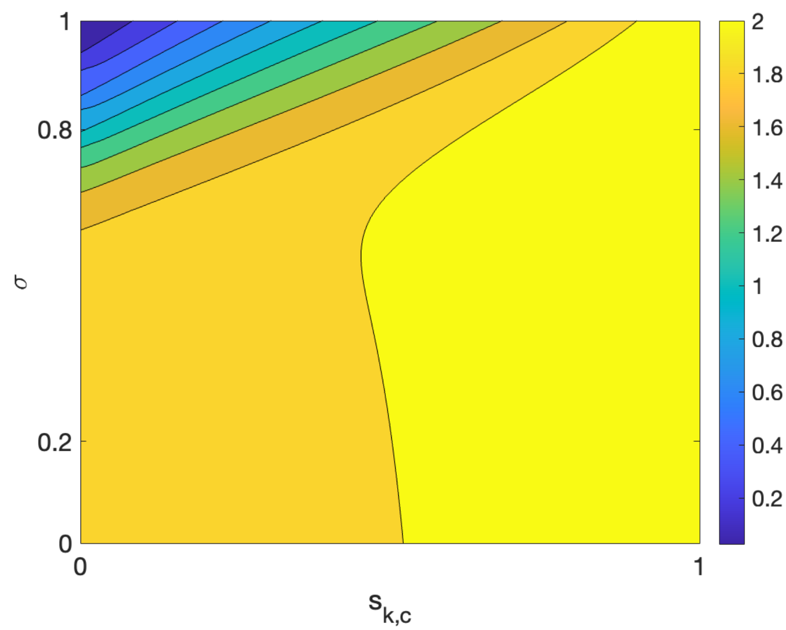

Figure 3: Level of inequality, computed as $(W_{c,t} - W_{w,t})/W_{w,t}$, $t = 1000$. Parameter values: $\delta = 0.03$, $\beta = \gamma = 0.9$, $\phi = 0.95$, $s_{u,c} = 1 - s_{k,c}$, $s_{u,w} = 0.6 s_{u,c}$

However, inequality increases when capitalists invest heavily in productive capital. This is because the interest paid to unproductive capital is linked to that of productive capital, and further growth of the latter results in even greater profits generated by unproductive capital (capitalists invest more than workers). ■

In the case where ϕ is less than zero, the fixed point E may be unstable (so the trajectories move away from it), it may be an improper poverty trap (as defined previously, i.e., a saddle) or a proper poverty trap, i.e., a stable fixed point. Once again, stability depends on the savings rate of capitalists ($s_{k,c}$), on the characteristics of the economy (identified by the elasticity of substitution ρ), and on the value of the ϕ parameter.

Proposition 2. *A "proper" poverty trap (i.e., E stable fixed point) exists in case of ϕ lower than zero. Developing countries are least susceptible to the poverty trap. In contrast, underdeveloped and developed countries might fall into it if the savings rate of capitalists ($s_{k,c}$) and the rate of return (ϕ) are low enough.*

Proof. Let us offer the following three-dimensional Figure 4 where the variety of values of the crucial parameters $\phi, \rho, s_{k,c}$ is represented and, therefore, shows the region of the poverty trap.

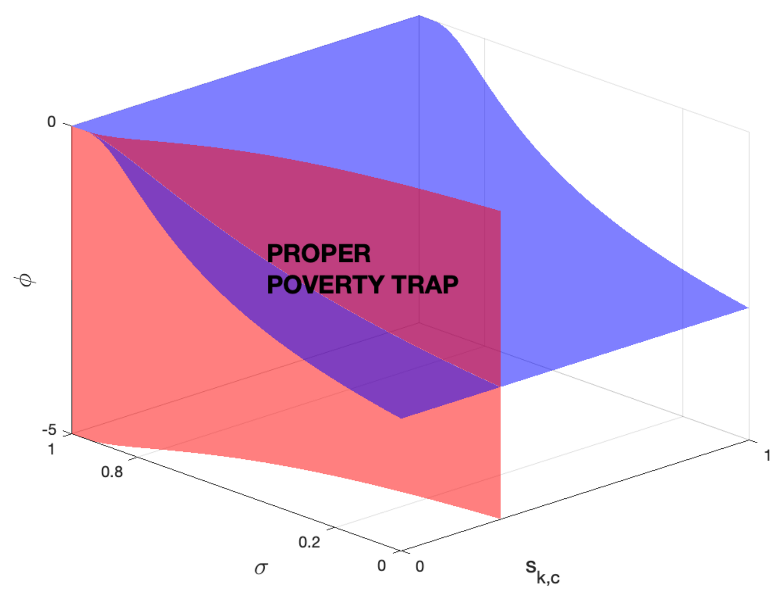

Figure 4: Region of the proper poverty trap moving $s_{k,c}$, σ and ϕ. The red surface is such that $s_{k,c} = \frac{\delta\beta^{\frac{1}{\rho}}}{\gamma}$ while the blue surface is such that $\phi = -\frac{2\beta^{\frac{1}{\rho}}}{\gamma}$. Parameter values: $\delta = 0.2$, $\beta = \gamma = 0.7$

Figure 4 shows the region of existence of the poverty trap for different values of the parameters ϕ, $s_{k,c}$ and σ. Our model map (11) shows that a stable (proper) poverty trap exists if two conditions are fulfilled simultaneously. The first condition is $s_{k,c} < \frac{\delta\beta^{\frac{1}{\rho}}}{\gamma}$ while the second condition is $\phi > -\frac{2\beta^{\frac{1}{\rho}}}{\gamma}$. Figure 4 draws the values of σ, ϕ, and $s_{k,c}$ on the three axes of the diagram. Now, let us discuss the conditions. First we know that it must be fulfilled $s_{k,c} < \frac{\delta\beta^{\frac{1}{\rho}}}{\gamma}$. The red surface is the one for which $s_{k,c} = \frac{\delta\beta^{\frac{1}{\rho}}}{\gamma}$ therefore to have the proper poverty trap, for this condition, we can only take the values of σ, ϕ and $s_{k,c}$ that are to the left of the red surface. But this is not enough because we also have the second condition that must be met, i.e. $\phi > -\frac{2\beta^{\frac{1}{\rho}}}{\gamma}$. The blue surface is such that $\phi = -\frac{2\beta^{\frac{1}{\rho}}}{\gamma}$ so the condition is fulfilled only above the blue surface. Therefore, the proper poverty trap region is the small region to the left of the red surface and above the blue surface. As a result, developing countries seem less susceptible to the poverty trap. For σ greater than 0.8, the poverty trap is stable (proper poverty trap) only for very low values of $s_{k,c}$ together with ϕ values close enough to zero. The area of existence of the poverty trap expands as the value of σ decreases, which implies higher saving rates (higher values of $s_{k,c}$) and lower rates of return, ϕ, on non-productive capital. ■

When ϕ is less than zero, there is also a high-level positive equilibrium, whose characteristics will be discussed in the next subsection.

3.2. Existence of the positive equilibrium - deterministic skeleton of the economy

In this section, we analyze the existence of a positive fixed point of the map (11), which is an equilibrium in which productive capital accumulation and the wealth of workers and capitalists assume positive values (see Appendix A for the demonstration of existence and stability of such a fixed point). Such an equilibrium exists only when $\phi < 0$, which means that the rate of return on unproductive capital is proportionally negative concerning the return on productive capital. We denote such a high-level equilibrium by,

$$E \equiv (K^*, W^*_{uc}, W^*_{uw})$$

where:

$$K^* = \left[\frac{(s_{k,c}\gamma\beta\delta^{-1})^{\frac{\rho}{1+\rho}} - \beta}{1-\beta}\right]^{\frac{1}{\rho}}, \quad W^*_{uc} = -s_{u,c}K^*\left(\frac{1}{\phi} + \frac{\delta}{s_{k,c}}\right), \quad W^*_{uw} = \frac{s_{u,w}(\beta-1)}{\beta\,(K^*)^{-(1+\rho)}}\left(\frac{1}{\phi} + \frac{\delta}{s_{k,c}}\right).$$

The wealth of capitalists and workers in equilibrium are:

$$W^*_c = K^*\left[\frac{\delta(1-s_{u,c})}{s_{k,c}} + 1 - \frac{s_{u,c}}{\phi}\right], \quad W^*_w = \frac{(1-\beta)(K^*)^{1+\rho}}{\beta}\left[\frac{\delta(1-s_{u,w})}{s_{k,c}} - \frac{s_{u,w}}{\phi}\right].$$

Proposition 3. *The high-level equilibrium $E \equiv (K^*, W^*_{uc}, W^*_{uw})$ is stable if:*

$$\frac{\delta}{\gamma}\beta^{\frac{1}{\rho}} < s_{k,c} < \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}\left[1 - \frac{2}{\delta(1+\rho)}\right]^{-\frac{1+\rho}{\rho}} \wedge s_{k,c} > -\frac{\phi\delta}{2}.$$

Proof. See Appendix A for the analytical proof. Figure 5 shows the three-dimensional region where the positive fixed point is stable in the parameter plane $(s_{k,c}, \sigma, \phi)$.

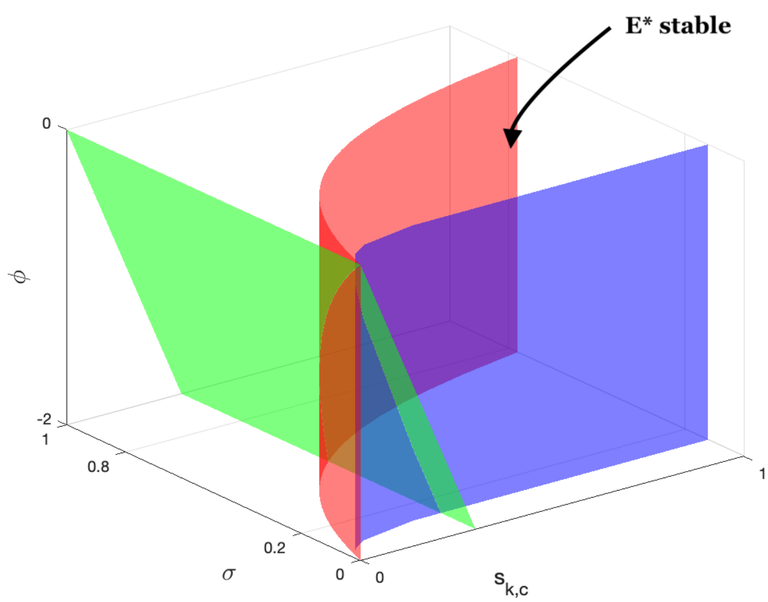

Figure 5: Region in which E^* is stable: between the red ($\frac{\delta}{\gamma}\beta^{\frac{1}{\rho}} < s_{k,c}$) and blue ($s_{k,c} < \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}\left[1 - \frac{2}{\delta(1+\rho)}\right]^{-\frac{1+\rho}{\rho}}$) surfaces, above the green ($s_{k,c} > -\frac{\phi\delta}{2}$) surface. Parameter values: $\gamma = \beta = 0.7$, $\delta = 0.2$, $s_{u,c} = 1 - s_{k,c}$, $s_{u,w} = 0.6 s_{u,c}$.

The fixed point E^* is stable only for values of ϕ, σ, and $s_{k,c}$ that lie simultaneously: 1) to the right of the green surface, 2) below the red surface, 3) above the blue surface. Notably, the equilibrium is stable mainly for developed economies ($\sigma \in (0.2, 0.8)$) and only when the capitalists' saving rate is sufficiently high. ■

We continue the analysis by studying the level of inequality between the wealth of the capitalists and that of the workers for the case in which the equilibrium is stable.

Proposition 4. *There could be a high-level stable equilibrium, mainly for developed economies. In such an equilibrium, inequalities are high when capitalists allocate their wealth primarily to unproductive capital assets (i.e., for low values of $s_{k,c}$). Otherwise, inequalities decrease.*

Proof. As seen in Figure 6, when positive equilibrium is reached[9] the wealth of the capitalists is greater than the wealth of the workers.

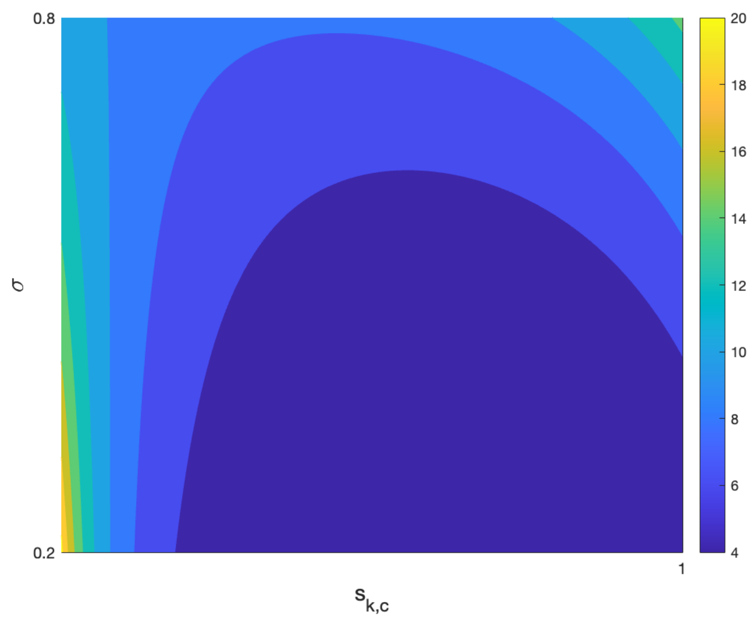

Figure 6: Contour lines of $\frac{W_c^* - W_w^*}{W_w^*}$ in the parameter plane $\sigma, (s_{k,c})$. Parameter values: $\gamma = \beta = 0.75$, $\delta = 0.2$, $\phi = -0.2$, $s_{u,c} = 1 - s_{k,c}$, $s_{u,w} = 0.6 s_{u,c}$.

Inequalities are predominant when capitalists allocate their resources mainly to unproductive capital (i.e., $s_{k,c}$ is low); in this case, the wealth gap (wealth concentration) reaches

[9]On the y-axes of Figure 6 there is σ ranging from 0.2 to 0 .8, therefore, is related to the developed economies, because as we wrote before the developed economies reach such an equilibrium.

the highest values. Conversely, when capitalists allocate their income mainly to productive capital ($s_{k,c} \to 1$), wealth inequality reaches the lowest values, especially for sufficiently low σ (recall that the threshold value $\sigma = 0.8$ divides countries into developed (below 0.8) and developing (above 0.8). ■

Proposition 5. *The stability of the high-level equilibrium depends on $s_{k,c}$, while it is not affected by $s_{u,c}$ or $s_{u,w}$. Economies would experience fluctuations when $s_{k,c}$ is excessively low. Developing and developed countries would evolve towards such an equilibrium for sufficiently high values of $s_{k,c}$. However, the most critical situation occurs in underdeveloped countries since fluctuations can arise when $s_{k,c}$ is excessively high due to oscillations generated in the productive capital market (boom and bust periods in production).*

Proof. The loss of stability may arise through a fold bifurcation in case $s_{k,c} < \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}$, while for $s_{k,c} > \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}\left[1 - \frac{2}{\delta(1+\rho)}\right]^{-\frac{1+\rho}{\rho}}$ or $s_{k,c} < -\frac{\phi\delta}{2}$ fluctuations appear (see Appendix A for the proof). Neither $s_{u,c}$ nor $s_{u,w}$ affect the system's stability. Conversely, the bifurcation curves are influenced by parameter ρ, which determines the elasticity of substitution σ; therefore, we investigate the bifurcation curves moving $s_{k,c}$ and σ. Note that bifurcation curves are lines in the parameter space in which the three eigenvalues of the system λ_1, λ_2 and λ_3 are non-hyperbolic (for the analytical expression of the eigenvalues see Appendix A).

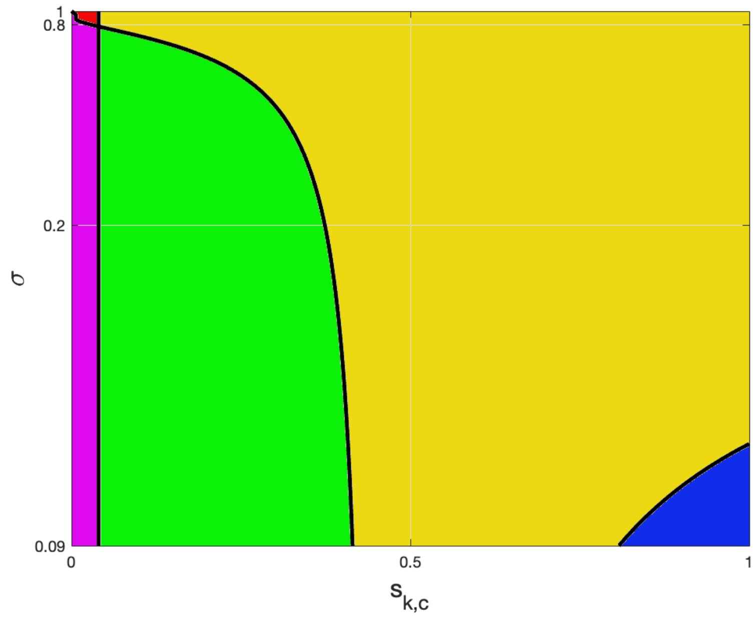

Figure 7: Bifurcation curves moving $s_{k,c}$ and σ. Red region: saddle with $|\lambda_1| < 1, \lambda_2, \lambda_3 < -1$. Pink region: unstable node with $\lambda_1 > 1, \lambda_2, \lambda_3 < -1$. Green region: saddle with $\lambda_1 > 1, \lambda_2, \lambda_3 \in (0, 1)$. Yellow region: stable node. Blue region: saddle with $\lambda_1 < -1, \lambda_2, \lambda_3 \in (0, 1)$. Parameter values: $\gamma = 0.9$, $\beta = 0.5$, $\delta = 0.4$, $\phi = -0.2$.

As noticed in Figure 7 all economies would experience fluctuations $(\lambda_2, \lambda_3 < -1)$ when $s_{k,c}$ is extremely low. For example, the fluctuations arise in the blue region, which means that the fluctuations arise because both $s_{k,c}$ is very high and σ is very low (that is, underdeveloped countries in which capitalists invest almost all in productive capital). Other fluctuations arise in the pink region for very low values of $s_{k,c}$ and for almost any value of σ (this means that in all types of economies (σ does not matter) if the capitalists invest very little in productive capital (fluctuations arise). For high values of $s_{k,c}$, developed and developing countries would always reach equilibrium (the fixed point is stable). In the case of underdeveloped economies ($\sigma \leq 0.2$) fluctuations can arise when $s_{k,c}$ is excessively high ($\lambda_1 < -1$). Once again, the role played by $s_{k,c}$ is key: regardless of the type of economy analyzed, low levels of investment in productive capital by capitalists can result in fluctuations and riddles generated by oscillations in the unproductive capital. Furthermore, in the case of underdeveloped countries, fluctuations could also arise if $s_{k,c}$ is too high. In this case, the fluctuations would be generated in the productive capital market (boom and bust production periods) and then spread to the entire economy. ■

The analysis conducted so far refers to the deterministic model, that is, without the stochastic component related to the profitability of unproductive capital. This will be discussed in the next section.

3.3. The stochastic case

The original model proposed in this paper considers the rate of return on unproductive capital given by

$$r_{u,t} = \phi r_{k,t} + \varepsilon_t$$

where $\varepsilon_t \approx N(0, \sigma^2)$ is white noise with variance σ^2. In the previous sections, we studied the results for the case of $\sigma^2 = 0$, obtaining the deterministic skeleton of the model. In this section, following Anufriev et al. (2020), we replicate the statistical properties of financial market returns with the stochastic version of the model. We cannot say anything analytically about the model in this stochastic case. Therefore, it must be calibrated to shed some light on the behavior of the main parameters that govern the economy. To this aim, we start by considering the main properties of a financial index that could represent the unproductive capital asset. Let's then consider the S&P GLOBAL LUXURY INDEX for the last 10 years. Focusing on daily observations of the index, see Figure 8; we see that some of the stylized facts that generally apply to asset returns do not apply to the Global Luxury Index: although the returns do not follow a Normal distribution (see also Table 1), the *volatility clusters* are not as evident in the time series of the returns and this is confirmed statistically by the sample autocorrelation functions that show a poor correlation in the squared returns and absolute returns.

Statistics	*Average*	*Min*	*Max*	*Volatility*	*Skewness*	*Kurtosis*
S&P Global Luxury Index	3.8885e-04	-0.11643	0.11243	1.5144e-04	-0.31638	13.6093

Table 1: Statistics related to the S&P GLOBAL LUXURY INDEX

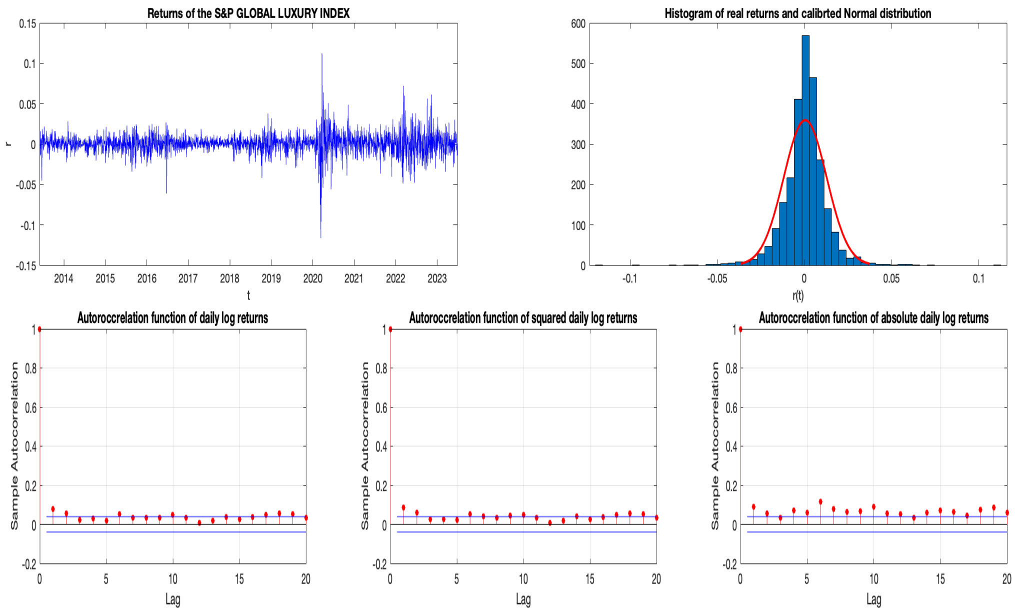

Figure 8: Top left panel: dynamics of daily returns of the S&P Global Luxury index over the period 31/05/2013-26/06/2023. Top right panel: histogram of the returns for the S&P Global Luxury index and normal distribution (red line) calibrated using the daily returns of the S&P Global Luxury index. Bottom left panel: autocorrelation function of the logarithmic daily returns of the S&P Global Luxury index. Bottom center panel: autocorrelation function of the squared logarithmic daily returns of the S&P Global Luxury index. Bottom right panel: autocorrelation function of the absolute logarithmic daily returns of the S&P Global Luxury index.

For the mapped model that we built, the objective is to replicate the characteristics of the distribution of returns on unproductive capital, for which we consider the S&P Global Luxury Index as a proxy. The index did not show dependence in the returns, quantitatively indicating the absence of the volatility clustering effect. With our model, we contribute to providing a qualitative explanation of the empirical distributions of returns of the non-productive assets further to investigate the effect of stochastic noise on inequalities. Within reach, we calibrated the stochastic version of our model to replicate the time series characteristics of the returns generated by the index.

As shown in Figure 9, the stochastic model allows replicating the trend of the index, and, in particular, no volatility clusters are evident even in the case of simulated returns, as shown by the autocorrelation functions. However, the simulation cannot capture outliers of returns such as those that occurred during the COVID-19 pandemic (2020) or the recent

Russo-Ukrainian conflict (2022). This can be seen in the trend of the two-time series (upper left panel of Figure 9) that do not overlap during the periods above and is reflected in the distribution of simulated returns whose values are closer to a normal distribution than those of the S&P Global Luxury index (upper right panel of Figure 9 and Table 2). Despite the differences between the observed and simulated returns, we can consider the model a good approximation of the index for medium and low market stress periods.

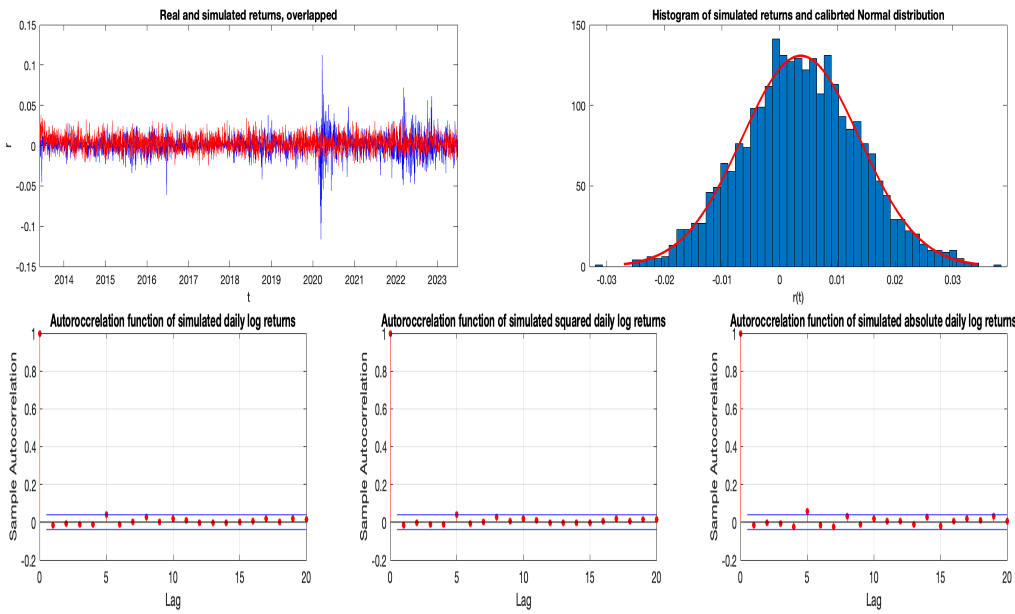

Figure 9: Top left panel: dynamics of daily returns of the S&P Global Luxury index over the period 31/05/2013-26/06/2023 (blue) and simulated returns (red). Top right panel: histogram of the simulated returns and normal distribution (red line) calibrated using the simulated returns. Bottom left panel: autocorrelation function of the logarithmic simulated returns. Bottom center panel: autocorrelation function of the squared logarithmic simulated returns. Bottom right panel: autocorrelation function of the absolute logarithmic simulated returns. Parameter values: $\beta = \gamma = 0.7$, $\delta = 0.01$, $\sigma = 0.82$, $s_{k,c} = 0.45$, $\phi = 0.15$, $\sigma_\varepsilon^2 = 0.0001$.

Statistics	*Average*	*Min*	*Max*	*Volatility*	*Skewness*	*Kurtosis*
Simulated returns	3.6359e-03	-0.03137	0.03813	1.0472e-04	0.04642	2.9221

Table 2: Statistics related to the simulated returns

Therefore, the simulation is performed considering the values $\beta = \gamma = 0.7$, $\delta = 0.01$, $\sigma = 0.82$, $s_{k,c} = 0.45$, $\phi = 0.15$ and $\sigma_\varepsilon^2 = 0.0001$. This corresponds to the characteristics of an economy on the borderline between developed and developing ($\sigma = 0.82$), in which capitalists invest approximately half of their wealth in productive capital ($s_{k,c} = 0.45$) and, considering the deterministic skeleton of the model, represents the case in which the unique equilibrium is the poverty trap and is unstable, confirming that for these parameter values no cycles or fluctuations are possible.

While for the simulation of $r_{u,t}$ it is not necessary to determine the values of $s_{u,c}$ and $s_{u,w}$, then from the proposed simulation, we analyze how different parameters affect wealth inequality between workers and capitalists. For this purpose we set a share $m \in [0, 1]$ and assume $s_{u,w} = m \cdot s_{u,c}$. This means that for any given value of $s_{u,c}$ (i.e., of the proportion of resources that capitalists invest in the non-productive asset), workers invest in the same asset a share $m \cdot s_{u,c}$ and therefore a share less than or at most equal (in the case $m = 1$) to that of the capitalists.

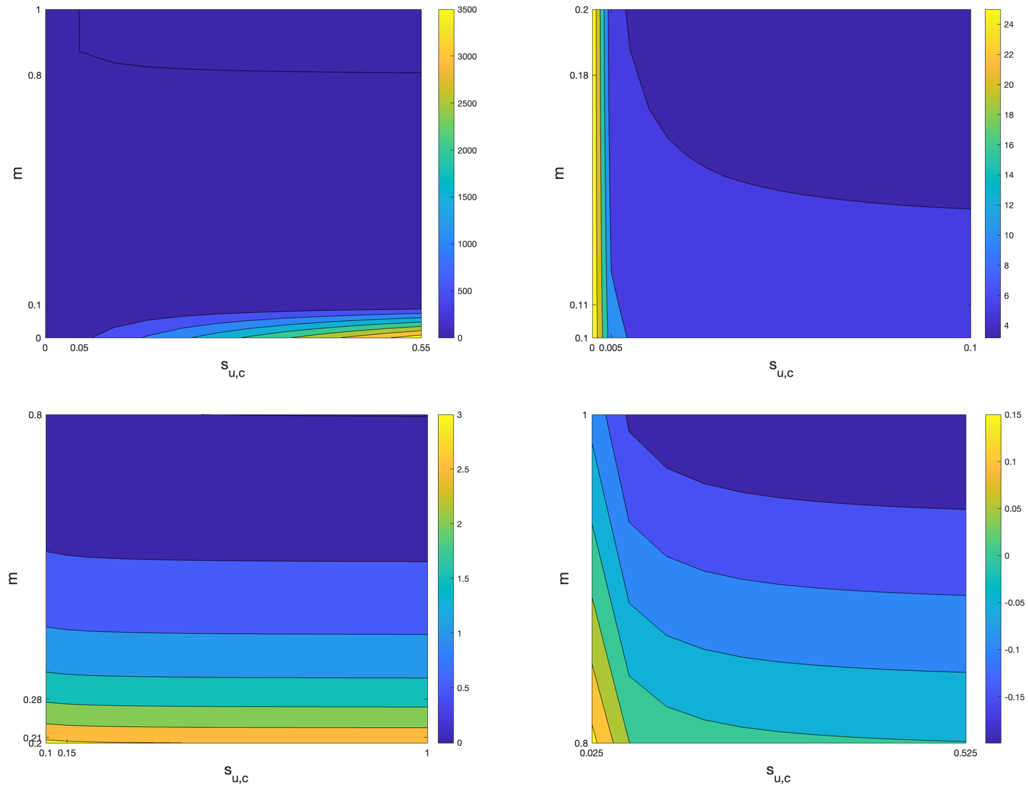

Figure 10: Top left panel: level of inequality (computed as $(W_c - W_w)/W_w$) in the long run for $m \in [0,1]$ and $s_{u,c} \in [0, 1 - s_{k,c}]$. Top right panel: enlargement for $m \in [0.1, 0.2]$, $s_{u,c} \in [0.005, 0.1]$. Bottom left panel: enlargement for $m \in [0.2, 0.8]$, $s_{u,c} \in [0.1, 1 - s_{k,c}]$. Bottom right panel: enlargement for $m \in [0.8, 1]$, $s_{u,c} \in [0.025, 1 - s_{k,c}]$. Parameter values: $\beta = \gamma = 0.7$, $\delta = 0.01$, $\sigma = 0.82$, $s_{k,c} = 0.45$, $\phi = 0.15$, $\sigma_\varepsilon^2 = 0.0001$, $s_{u,w} = m \cdot s_{u,c}$.

As visible in Figure 10, in the long run, the inequality is greater the higher the value of $s_{u,c}$ and the lower the value of m (top left panel). Although this result is predictable, new insights emerge, i.e.:

i) Even when capitalists invest very little in unproductive capital, the level of inequality is high (upper right panel). Therefore, the simulations show that in a sufficiently developed economy (σ =0.82) in which the performance of non-performing assets replicates that of the S&P Global Luxury Index and capitalists invest slightly less than half of their wealth in productive capital, in the long run, the highest levels of inequality between capitalists and workers occur when any agent (capitalists and workers) invests very little in unproductive capital,

ii) While inequality would approach zero if workers had enough resources for their livelihood to invest almost the same amount as capitalists in unproductive capital (bottom left panel of Figure 10) and for $s_{u,w} \to s_{u,c}$ workers might even have greater wealth than capitalists (bottom right panel of Figure 10).

Lemma 2. *In a sufficiently developed economy (σ =0.82) in which the performance of unproductive assets replicates that of the S&P Global Luxury Index, and capitalists invest slightly less than half of their wealth in productive capital, in the long run, the level of inequality would fall to zero if workers invest almost as much as capitalists in non-productive capital. The lower the welfare of workers (and therefore used for current consumption rather than investment), the higher the level of inequality between the two categories.*

What would happen to the level of inequality for lower or higher values of the saving rate of capitalists focused on productive capital ($s_{k,c}$)? As seen in the left panel of Figure 11, the higher the investment capitalists allocate to productive capital (the higher $s_{k,c}$), the lower the level of inequality over time.

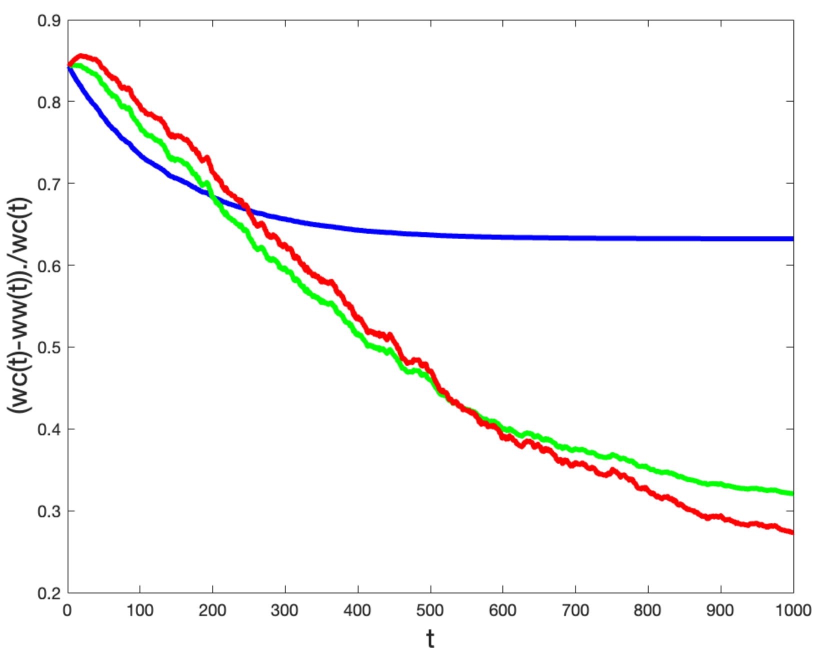

Figure 11: Evolution of inequality (computed as $(W_c - W_w)/W_w$) in the long run. Blue line $s_{k,c} = 0.2$, green line $s_{k,c} = 0.6$, red line $s_{k,c} = 0.8$. Parameter values: $\beta = \gamma = 0.7$, $\delta = 0.01$, $\sigma = 0.82$, $\phi = 0.15$, $\sigma_\varepsilon^2 = 0.0001$, $s_{u,c} = 0.2$, $m = 0.5$, $s_{u,w} = m \cdot s_{u,c}$.

In other words, following the previous results, the simulations have shown that inequality is reduced if workers can invest in unproductive capital assets; this must be a part of their wealth similar to that of the capitalists (m close to 1). This suggests that although the resources of workers are often inflexible because labor income is devoted mainly to consumption, and it would not be possible to implement policies that could increase their investment in non-productive capital (increase m), the role of capitalists is essential to achieve acceptable levels of inequality, as stated in the following statement.

From the above, we can affirm that economic policies that favor capitalists' investment in productive capital assets (increasing $s_{k,c}$) decrease inequality in the long run.

3.4. The case of Bubbly Episodes

Let us now continue with the case of bubbly assets. As we have introduced, the standard way in macroeconomics to treat unproductive assets is through the notion of "bubbles". Well-studied examples have been analyzed by the seminal research papers of Samuelson (1958), Tirole (1985), and Martin and Ventura (2012). Although an in-depth analytical study could be carried out in a later paper, we intend to simulate the economy's performance here, considering the case in which the return on unproductive capital is endogenously described and may be bubble-bearing. To do this, we base our analysis on the Martin and Ventura (2018) model and introduce the random creation and destruction of bubbles for the return of unproductive capital. Bubbles are assets that do not produce output, and their returns depend solely on future prices. We might think of bubbles attached to objects (such as luxury objects) as having a fundamental value F that does not change over time. However, its price P_t incorporates a bubble component b_t. From this it follows that the price of unproductive capital at $t+1$ is

$$P_{t+1} = F + \sum_{j=0}^{t+1} b_j$$

and consequently the return of the unproductive asset is

$$r_{u,t+1} = \frac{b_{t+1}}{F + \sum_{j=0}^{t} b_j}. \tag{12}$$

We assume that the economy transitions between three states of the world (the positive bubble state, the negative bubble state, the fundamental bubble state): $z_t \in \{PB, NB, FU\}$. During the Positive Bubble state, PB, new bubbles pop up with size $\mu > 0$; during the Negative Bubble state NB, a negative bubble arises with size $-\mu < 0$, while in the Fundamental state FU, no bubble appears. The three outcomes have probabilities p_1, p_2 and p_3 with $p_1 + p_2 + p_3 = 1$ and with this assumption we have

$$b_t = \begin{cases} \mu & \text{if } z_t = PB \\ -\mu & \text{if } z_t = NB \\ 0 & \text{if } z_t = FU \end{cases} . \tag{13}$$

Therefore, let us now numerically investigate the economy's evolution and inequality level when the return on non-performing assets is modeled as bubbles. As for the stochastic case in the previous section, Figure 12 presents the level of inequality in the long run for different scenarios: we study the case in which the market is very likely to show positive bubbles (most common case, $p_1 = 0.8$, $p_2 = p_3 = 0.1$), the case in which the unproductive asset is more often devalued ($p_2 = 0.8$, $p_1 = p_3 = 0.1$), the case in which the value of the unproductive asset remains usually close to its fundamental value ($p_3 = 0.8$, $p_1 = p_2 = 0.1$), and the case in which each scenario has the same probability ($p_1 = p_2 = p_3 = 1/3$).

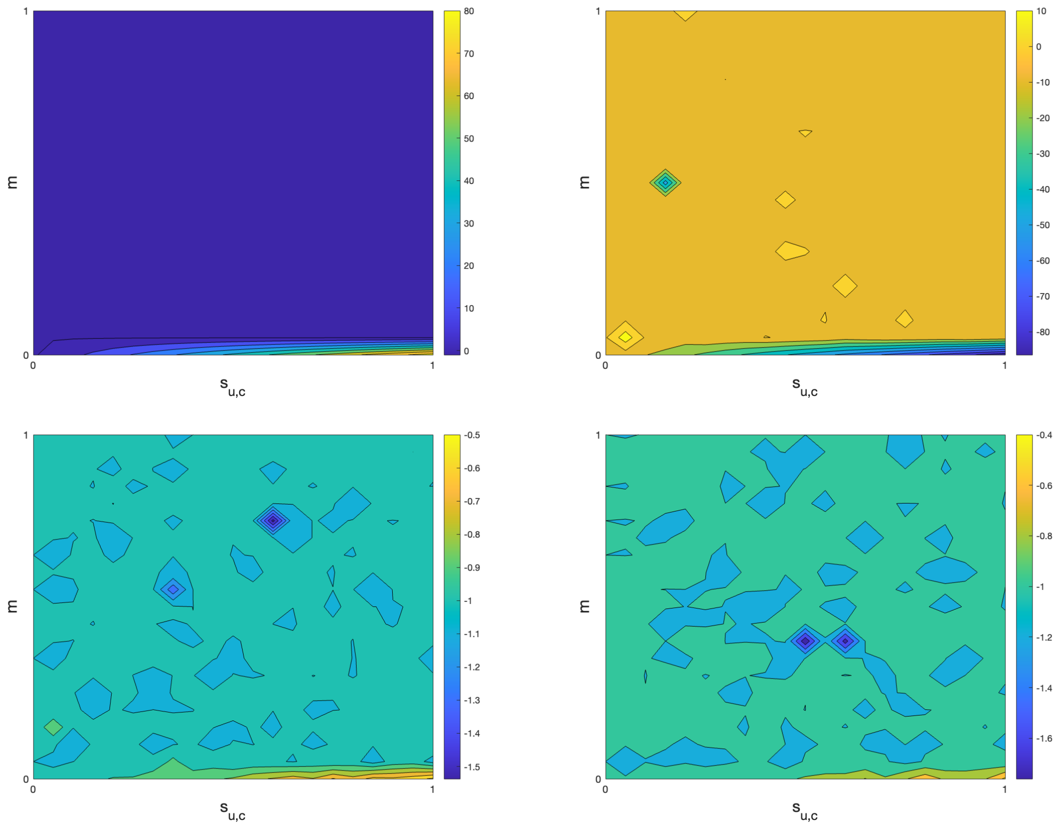

Figure 12: Level of inequality (computed as $(W_c - W_w)/W_w$) in the long run for $m \in [0,1]$ and $s_{u,c} \in [0, 1 - s_{k,c}]$. Top left panel: $p_1 = 0.8$, $p_2 = p_3 = 0.1$. Top right panel: $p_2 = 0.8$, $p_1 = p_3 = 0.1$. Bottom left panel: $p_3 = 0.8$, $p_1 = p_2 = 0.1$. Bottom right panel: $p_1 = p_2 = p_3 = 1/3$. Parameter values: $\beta = \gamma = 0.7$, $\delta = 0.01$, $s_{k,c} = 0.45$, $\sigma = 0.82$, $s_{u,w} = m \cdot s_{u,c}$. Average values after 1000 iterations.

We find the results presented for the case where the return on unproductive assets is calibrated considering the S&P Global Luxury Index still holds even if the return on unproductive assets is modeled as a positive bubble. In the rare case that the value of the unproductive asset is often underestimated, the results are the opposite and, in the long run, the welfare of the workers is even greater than that of the capitalists, especially if the workers do not invest much in such an asset (as evident). The lowest levels of inequality occur when the value of unproductive capital remains close to its fundamental value or has the same probability of being evaluated as a positive bubble, a negative bubble, or equal to its fundamental value. It should be noted, however, that in the main literature related to bubbles, these last cases are usually omitted. The most studied phenomenon is positive bubbles, in which case the results presented in the previous section continue to hold. That is, investing more in productive capital is equivalent to investing less in unproductive capital, whether for developed or developing economies, since this will significantly help reduce the concentration of wealth and avoid poverty traps and riddles and waves of economic growth.

4. Conclusion

We have developed a dynamic non-linear model in which the economy is populated by two types of agents: workers and capitalists. The former receive a wage for their work and can invest in a non-productive asset, while the latter earn income from production and can invest in both productive and non-productive assets. We investigated the deterministic skeleton of the model in which noise in asset returns is absent and found that a poverty trap could exist if capitalists invest too little in productive capital; furthermore, developing countries are the least prone to stagnation: they are least likely to fall into such a poverty trap, and their long-term level of inequality is low compared with undeveloped and developed economies alike. Then, we show that there is a positive high-level equilibrium, which is reached mainly by developed countries. Nevertheless, all economies experience fluctuations and riddles if capitalists do not invest sufficiently in productive assets. Even when the stochastic and bubbly component is considered, the investment level in productive capital remains decisive. We present numerous simulations that show that an increase in productive investment and a restriction of the levels of investment in non-productive capital allow the mitigation of inequality between workers and capitalists in the long term. The results hold even if the unproductive capital is modeled as a bubble in the sense of Tirole (1985) and Martin and Ventura (2018).

Furthermore, what drives our model is exogenous shocks to the aggregate supply of a bubble asset, which, in turn, become shocks to bubble prices and returns. These shocks would have no impact in a conventional DSGE model, but they do in ours because wealth accumulation makes capitalists less risk-averse, stimulates bubble demand, and slows growth.

Therefore, future research could present a Blanchard/Yaari-style toy model with infinite-life agents. It will have two assets: productive capital and an unproductive bubble with exogenous supply variations. The hard part will be to make risk aversion (or intertemporal elasticity of substitution) sensitive to household wealth.

The arguments of this paper suggest that economies manage to avoid poverty traps and obtain a reduction in wealth inequality among their agents when economic policies that favor investment in productive capital are implemented. Investing in productive capital instead of non-productive capital removes economies from the threat of a poverty trap and reduces wealth inequality in the long run. We leave this argument for implementing such economic policies as future research to analyze the role of the policy maker in this non-linear macroeconomic model that we have developed. That is, the agent called the government and its implementation of economic policies that favor or not the greater productivity of capital in the economies.

Appendix A. Deterministic skeleton of the model

The deterministic model is obtained from map T given by (11) setting $\sigma_\varepsilon^2 = 0$. The resulting system is given by

$$T_d = \begin{cases} K_{t+1} = & (1-\delta)K_t + s_{k,c}\gamma\beta\left[1+\beta\left(K_t^{-\rho}-1\right)\right]^{-\frac{1+\rho}{\rho}} K_t^{-\rho} \\ W_{uc,t+1} = & (1+\phi r_{k,t+1})\left\{W_{uc,t} + s_{u,c}\gamma\beta\left[1+\beta\left(K_t^{-\rho}-1\right)\right]^{-\frac{1+\rho}{\rho}} K_t^{-\rho}\right\} \\ W_{uw,t+1} = & (1+\phi r_{k,t+1})\left\{W_{uw,t} + s_{u,w}\gamma(1-\beta)[1+\beta(K_t^{-\rho}-1)]^{-\frac{1+\rho}{\rho}}\right\} \end{cases} \tag{A.1}$$

subject to

$$W_{uc,0} = 0, \qquad W_{uw,0} = 0.$$

Equilibria of map T_d obtained setting $K_{t+1} = K_t = K$, $W_{uc,t+1} = W_{uc,t} = W_{uc}$ and $W_{uw,t+1} = W_{uw,t} = W_{uw}$ in (A.1).

The trivial equilibrium

$$E \equiv (0,0,0)$$

always exists. Moreover, a non-negative solution exists for $\phi < 0$ and is given by

$$E^* \equiv (K^*, W_{uc}^*, W_{uw}^*)$$

with

$$K^* = \left[\frac{\left(s_{k,c}\gamma\beta\delta^{-1}\right)^{\frac{\rho}{1+\rho}} - \beta}{1-\beta}\right]^{\frac{1}{\rho}}, \quad W_{uc}^* = -s_{u,c}K^*\left(\frac{1}{\phi} + \frac{\delta}{s_{k,c}}\right), \quad W_{uw}^* = \frac{s_{u,w}(\beta-1)}{\beta\left(K^*\right)^{-(1+\rho)}}\left(\frac{1}{\phi} + \frac{\delta}{s_{k,c}}\right).$$

The Jacobian matrix of system (A.1) is

$$J = \begin{bmatrix} \lambda_1 & 0 & 0 \\ A & \lambda_2 & 0 \\ B & 0 & \lambda_3 \end{bmatrix}$$

where

$$\lambda_1 = 1 - \delta + s_{k,c}\gamma\beta\frac{\beta - (1-\beta)\rho K^\rho}{[\beta + (1-\beta)K^\rho]^{\frac{1+2\rho}{\rho}}},$$

$$\lambda_2 = \lambda_3 = 1 + \phi\gamma\beta[\beta + (1-\beta)K^\rho]^{-\frac{1+\rho}{\rho}}.$$

Considering the trivial fixed point $E \equiv (0,0,0)$, the eigenvalues computed at the equilibrium are

$$\lambda_1(E) = 1 - \delta + s_{k,c}\gamma\beta^{-\frac{1}{\rho}}, \quad \lambda_2(E) = \lambda_3(E) = 1 + \phi\gamma\beta^{-\frac{1}{\rho}}. \tag{A.2}$$

The first eigenvalue loses stability via fold bifurcation in case $s_{k,c} > \frac{\delta\beta^{\frac{1}{\rho}}}{\gamma}$. The two coincident eigenvalues may lose stability via fold bifurcation in case $\phi > 0$ and via flip bifurcation for $\phi < -\frac{2\beta^{\frac{1}{\rho}}}{\gamma}$.

Considering the non-trivial equilibrium $E^* \equiv (K^*, W^*_{uc}, W^*_{uw})$, the eigenvalues evaluated at the fixed point are

$$\lambda_1(E^*) = 1 + \delta(1+\rho)\left[\left(\frac{\delta^\rho\beta}{s^\rho_{k,c}\gamma^\rho}\right)^{\frac{1}{1+\rho}} - 1\right]$$

$$\lambda_2(E^*) = \lambda_3(E^*) = 1 + \frac{\phi\delta}{s_{k,c}}$$

The fixed point is stable if all the eigenvalues have magnitude less than one.

The first eigenvalue may lose stability via fold bifurcation ($\lambda_1(E^*) > 1$) in case $s_{k,c} < \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}$ and via period doubling bifurcation ($\lambda_1(E^*) < -1$) in case $s_{k,c} > \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}\left[1 - \frac{2}{\delta(1+\rho)}\right]^{-\frac{1+\rho}{\rho}}$. The two coincident eigenvalues $\lambda_2(E^*), \lambda_3(E^*)$ lose stability only via flip bifurcation ($\lambda < -1$) when $s_{k,c} < -\frac{\phi\delta}{2}$ (recall that the fixed point exists for $\phi < 0$).

These results can be summarized as follows:

- for $\phi \geq 0$ a unique fixed point exists, given by E;
  - E is a saddle in case $s_{k,c} \leq \frac{\delta\beta^{\frac{1}{\rho}}}{\gamma}$
  - E is an unstable node in case $s_{k,c} > \frac{\delta\beta^{\frac{1}{\rho}}}{\gamma}$
- for $\phi < 0$ two fixed points exist: E and the non-trivial equilibrium E^*;

- for $s_{k,c} < \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}$
  * E is stable in case $\phi > -\frac{2\beta^{\frac{1}{\rho}}}{\gamma}$ and a saddle otherwise.
  * E^* is a saddle in case $\phi > -\frac{2s_{k,c}}{\delta}$ and unstable otherwise
- for $\frac{\delta}{\gamma}\beta^{\frac{1}{\rho}} < s_{k,c} < \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}\left[1 - \frac{2}{\delta(1+\rho)}\right]^{-\frac{1+\rho}{\rho}}$
  * E is a saddle in case $\phi > -\frac{2\beta^{\frac{1}{\rho}}}{\gamma}$ and unstable otherwise.
  * E^* is stable in case $\phi > -\frac{2s_{k,c}}{\delta}$ and a saddle otherwise
- for $s_{k,c} > \frac{\delta}{\gamma}\beta^{\frac{1}{\rho}}\left[1 - \frac{2}{\delta(1+\rho)}\right]^{-\frac{1+\rho}{\rho}}$
  * E is a saddle in case $\phi > -\frac{2\beta^{\frac{1}{\rho}}}{\gamma}$ and unstable otherwise.
  * E^* is a saddle in case $\phi > -\frac{2s_{k,c}}{\delta}$ and unstable otherwise.

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