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How Emissions Charges affect Multiple equilibria and Stability in a Dynamic Monopoly

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Abstract

Climate change has directed policymakers' focus to the impact of economic policies on global emissions. And there is a urgent need for environmental policies to reduce emissions. This paper explores the effects of emission charges within a monopoly framework where the demand function is unknown. Unlike traditional analyses, this research examines how emission charges, multiple equilibria, and their stability affect emissions reduction policies, particularly in scenarios involving non-linear dynamics and initial conditions.

We investigate a monopolist introducing a new product, who learns the demand function using a rule of thumb. This approach reveals that while emission charges generally reduce production and emissions, they can also lead to multiple equilibria - two maxima and a minimum - depending on demand and cost parameters. The stability of these equilibria is influenced by initial conditions and the firm's response to marginal returns. High taxation can destabilize production, resulting in fluctuating emissions.

This research contributes to the theoretical literature on environmental governance by emphasizing the importance of non-linearity and the unknown demand function in shaping policy outcomes. Also through numerical analysis, we show that learning processes significantly impact the stability of multiple equilibria, highlighting the complexities in designing effective environmental policies in real-world economic contexts.

Keywords: Emission charges; Climate Change; Complex Dynamics; Monopoly; Multiple Equilibria; Stability

JEL classification: C62, D42, Q51, Q58

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1 Introduction

Climate change is focusing policymakers' attention toward the role that various economic policy instruments can play in influencing global emissions. According to the IPCC report (2022), the adoption of environmental policies aimed at reducing production and emissions is urgent. At the same time, the role of non-linearities in explaining the possible non-trivial effects of environmental policies on the production system is relevant.

To the best of our knowledge, the relationship between emission charges, multiple equilibria and how their (in)stability may (or may not) affect emission reduction policy has not been well studied in a monopoly framework with an unknown demand function. We believe that it certainly deserves maximum attention, as the dynamics, through the presence of positive and negative feedbacks, can influence the final outcome of a given economic policy. Non-trivial unexpected results could arise from specific initial conditions as well as from complex dynamics.

The literature on the effects of emissions charges on production levels and global emissions is now mature. In this paper we want to focus on the effects of emissions charges when the demand function is unknown and there is the possibility of multiple equilibria. Specifically, we consider the case where a new good enters the market and the only producer (monopolist) does not know the demand function but learns it by a rule of thumb. Furthermore, we focus on the possibility that the demand function has characteristics that lead to multiple equilibria. A monopoly, although simple, can provide us with different and clear insights. Moreover, precisely because of its simplicity, the finding of non-trivial effects within this framework leads us to be cautious in a more complicated economic environment, where the interactions between consumers and firms play an additional role.

We find, as expected, that the increase in emissions charges leads to a reduction in production and hence in emissions themselves. However, in the presence of an increasing segment of the marginal revenue curve, two other stationary states can emerge, therefore leading to the coexistence of two maxima and a minimum. While the minimum is always unstable, the stability of the two maxima depends on the cost and demand parameters and the firm's response to marginal profits. The minimum is the separating value between two attractors. This means that the strength of the anti-emissions policy depends crucially on the initial conditions. At the same time, too high a level of taxation leads to instability of the maxima, making production (and therefore emissions) unstable. Finally, the two attractors merge and production moves between a minimum and a maximum, leading also to strong fluctuations in emissions.

Our paper lies in a wider theoretical and empirical literature in economics on environmental governance and climate action with an dynamical approach (see Costanza et al., 1993 and Bargigli and Ricchiuti, 2022). Methodologically, it relates on the role of non-linearity and unknown demand function in shaping environmental policies. In the third chapter of her well-known book on Imperfect Competition, Joan Robinson discusses the case in which there are multiple equilibria in the monopoly framework (Robinson, 1969, pp.57). She states that: *'Cases of multiple equilibrium may arise when the demand curve changes its slope, being highly elastic for a stretch, then perhaps becoming relatively inelastic, then elastic again. This may happen, for instance, in a market composed of several groups of consumers each with a different level of incomes. [...]'*. The demand function that Robinson has in mind is such that the corresponding marginal revenue has an increasing trait between two decreasing traits. Walters (1980) studies the case of an increasing marginal revenue when there is a convex kinked demand curve. While Formby et al. (1982) show that the condition for an increasing marginal revenue are not at all stringent and, therefore, the existence of multiple equilibria can be pervasive. However, they both show these conditions with numerical examples.

In the same pages Joan Robinson discussed the possibility that consumers do not know the entire demand function, highlighting that: *'it is natural to object that this method of analysis is highly artificial. Of what use, the reader may ask, to discuss fine points of analysis which depend upon the shapes of demand curves when no everyday monopolist has any such ideas in his mind, and when even the most up-to-date business have only the vaguest notion of what kind of demand curves they have to deal with? [...]'* And again: *'We need not imagine that he is able to plot the demand and cost curves throughout their length, but merely that he can see whether selling a little more of his product than he does at present will increase or decrease his net gains. As long as marginal revenue exceeds marginal cost, there will be a tendency for him to increase output, and as long as marginal revenue falls short of marginal cost, there will be a tendency for him to contract output, and he will be in equilibrium at the monopoly point.'* To the best of our knowledge, Clower (1959, p. 705) firstly highlights the importance of *"introduce [...] complications into traditional models of price and output determination"*, assuming, within the monopoly mathematical framework, that consumers have no information on the demand function.

Two different lines of research have investigated the agents' behavior when the demand function is unknown. Within the first approach, firms form conjectures analyzing the interaction between past decisions and market mechanisms. This research's line started with the seminal paper of Negishi (1961). He introduces, in a general equilibrium model, imperfect competitors who act as price maker; assuming that monopolistically competitive firms deal with *subjective* inverse demand (supply) functions for their output (inputs), they conjecture that the demand is a decreasing linear function of the price and pass through the observed status quo. Silvestre (1977) adds an hypothesis in order to link *subjective* and *objective* demand function: the slope of the former demand curve coincides with the slope of the latter, meaning that firms know the *current* elasticity. While this approach focuses mainly on the existence of equilibria, it fails to consider the crucial problem of determining how agents coordinate each other in order to reach the optimal choice. Recently, several authors (Leonard and Nishimura, 1999; Tuinstra, 2004; Naimzada and Sbragia, 2007, Naimzada and Ricchiuti, 2011) have analyzed decision making processes when agents have bounded rationality and the rational choice emerges through the dynamical processes of selection, adaptation and learning (Alchian, 1950).

The second approach assumes that firms try to extrapolate information using simple rules of thumb, in line with Robinson's discussion reported above. Baumol and Quandt (1964) firstly suggest, as a rule of thumb, the gradient rule within a monopoly model. In order to mimic the knowledge of the demand function, the monopolist expects that if there is a positive (negative) variation of profits, this could move the price in the same (opposite) direction as that of the previous period. Otherwise, if profits do not change, the monopolist will not change the price. This mechanism generates a steady state that is exactly the level of price that maximizes profits, as in classical microeconomic theory. Puu (1995) recovers their model in a discrete time setting employing a demand function that has an inflection point: a cubic demand. Naimzada and Ricchiuti (2008) show that complex dynamics can be achieved even if the demand function does not have an inflection point: it is sufficient an high enough reaction coefficient to profit variations.

Throughout the present paper, we use the second approach, assuming that the demand function is like the one studied by Puu and, in particular, focusing on the set of parameters such that the marginal revenue (as suggested by Joan Robinson) has an increasing trait.

These two elements allow us to better study the effects of environmental policy, as analyzed by Mamada and Perring (2020), both on the number of equilibria and on their stability, thus giving us some insights into the emission charges over time[1]. Mamada and Perrings (2020) study how emission charges affect market structure, output and emissions in Cournot competition. Assuming myopic decision making and partial matching, we find that increasing marginal emission charges stabilise duopolies, while decreasing charges destabilise them, leading to monopolies. Equilibrium output and emissions are higher under cost structures favouring monopolies than those favouring duopolies.

Matsumoto and Szidarovszky (2022) develop an n-firm Cournot oligopoly with a hyperbolic price function and use a game-theoretic approach to examine how environmental charges affect total and individual NPS pollution levels. The main findings are: (1) individual Cournot output is affected by marginal production cost, average abatement technology, and the number of firms; (2) environmental charges effectively control total NPS pollution when the average marginal production cost is less than the average emission coefficient; (3) environmental charges control individual pollution when the tax rate and the average pollution level are high enough; (4) stronger conditions are required for the effectiveness of environmental charges under hyperbolic demand compared to linear demand.

Naimzada and Pireddu (2023) extend the dynamic Cournot duopoly model with emission charges of Mamada and Perrings (2020) to differentiated goods and analyse static and dynamic results. Firms are taxed on their own emissions using quadratic charge functions, and adjust output partly due to capacity constraints. The steady state, which is consistent with the Nash equilibrium, is allowed if marginal emission charges are positive. Their results show that environmental policy is fully effective and globally stable for independent goods or goods with low interdependence, whether substitutes or complements. In an other paper, Naimzada and Pireddu (2024) replace the linear partial adjustment rule in Mamada and Perrings (2020) with a sigmoid adaptive best response mechanism in a Cournot duopoly model with quadratic emission charges and homogeneous goods. The sigmoid nonlinearity accounts for bounded output variations due to constraints and produces stable dynamic outcomes. They analyse the stability of the unique steady state (Nash equilibrium) and the impact of parameters on stability. Two comparative dynamics studies evaluate the effectiveness of environmental policy when the Nash equilibrium is unstable. The first shows the effectiveness of the policy for both complements and substitutes at different charge levels. The second reveals higher emissions along non-stationary trajectories compared to the equilibrium path, demonstrating that adjusting the sigmoid mechanism can reduce output fluctuations, stabilize the system and limit pollution.

The rest of the paper is organized as follows. In Section 2 we present the model and discuss the existence and stability of multiple equilibria. In Section 3, through numerical simulations, we show how the learning process affect the stability of equilibria. In Section 4 we discuss and report concluding remarks.

2 The model

Following the intuition of Robinson (1928) discussed above, we assume that the demand function is cubic (see Puu, 1990 and Naimzada and Ricchiuti, 2008), and it comes in the form:

$$P(q) = a - \frac{b}{2}q + \frac{c}{3}q^2 - \frac{d}{4}q^3 \tag{1}$$

where $q \geqslant 0$ is the output and a, b, c, d are positive constants. The corresponding revenue function is $R(q) = q \cdot P(q)$. It is easily seen that P is a strictly decreasing function on the positive axis

[1]The present paper will not address other mechanisms used by governments to deal with climate change, as Emission Trading Systems (see for example Antoci et al.,2022)

provided $c^2 < (27/8)bd$ and that the marginal revenue function R' has a local minimum and a local maximum, that is its graph displays an increasing trait, provided $c^2 > 3bd$ (see Figure 1). Since $3 < 27/8$, this means that we may ensure both features by selecting c in the interval:

$$\sqrt{3bd} < c < \sqrt{\frac{27}{8}bd}. \tag{2}$$

We next assume that production costs are quadratic in q and given by the function

$$C^p(q) = \frac{m}{2}q^2 \tag{3}$$

for a positive constant m.

Finally, the monopolist is subject to emission charges that are quadratic in the output (as in Mamada and Perrings, 2020; Naimzada and Pireddu, 2023), and given by the function

$$C^e(q) = ku + \frac{ks}{2}u^2 \qquad u = \varepsilon q \tag{4}$$

where the positive constants k, ε, s are the key parameters of the environmental policy. Indeed, here: k is the intensity of environmental policy, s determines the shape of C^e and ε gives the emission per unit output.

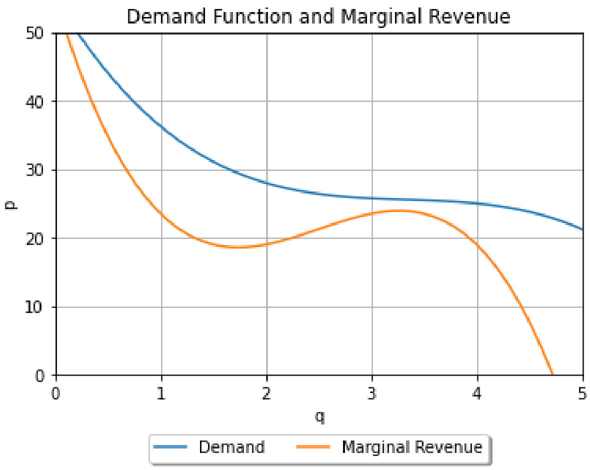


Figure 1: Downward demand function and marginal revenue with positive slope

The profit is then given by the quartic function

$$\pi(q) = q \cdot P(q) - C^p(q) - C^e(q) = (a - \varepsilon k)q - \frac{b + m + s\varepsilon^2 k}{2}q^2 + \frac{c}{3}q^3 - \frac{d}{4}q^4 \tag{5}$$

and the marginal profit π' is a cubic function, thus presenting the possibility of multiple equilibria.

We assume that the market is new and it needs to be explored, therefore the monopolist does not know the entire demand function. She uses a simple rule of thumb - a gradient rule - choosing the quantity to be produced (Baumol and Quandt, 1964; Puu, 1990; Bischi and Lamantia, 2002; Naimzada and Ricchiuti 2008). Over time, the production increases (decreases) if the marginal profit increases (decreases) and it is stable when the marginal profit is zero. This mechanism is captured by the following dynamics:

$$q_{t+1} = q_t + \gamma\pi'(q_t) \tag{6}$$

where the positive constant γ describes how the output reacts to marginal profit's variations. We immediately see that the steady states of the dynamics (6) are precisely the critical points of the profit function π, that is the points where the marginal profit π' vanishes.

2.1 Steady states and equilibria

In what follows, we discuss the existence and the number of equilibria for the dynamics (6), that are the positive zeros of the cubic function

$$\pi'(q) = a - \varepsilon k - (b + m + s\varepsilon^2 k)q + cq^2 - dq^3 \tag{7}$$

We may safely assume that when production is zero the marginal profit is positive, that is we posit $a > \varepsilon k$. In order to ease notation, we set

$$A = a - \varepsilon k \qquad B = b + m + s\varepsilon^2 k \qquad C = c \qquad D = d \tag{8}$$

so that

$$\pi'(q) = A - Bq + Cq^2 - Dq^3 \tag{9}$$

We observe that all four constants in (8) are positive, by the stated assumptions on the parameters. Therefore, by Descartes' Rule of Signs, the number of positive roots of the cubic function π' in (9), with properly counted multiplicities, is either 1 or 3. In other words, there exists at least one and at most three equilibria for the dynamics (6). Moreover, recalling that the discriminant of the cubic equation $\pi' = 0$ is defined as

$$\Delta = 18ABCD - 4AC^3 + B^2C^2 - 4B^3D - 27A^2D^2, \tag{10}$$

a classical result in Algebra (see Irving, 2004) states that the case $\Delta > 0$ corresponds to three distinct positive roots, while the case $\Delta < 0$ corresponds to just one positive root.[2]

Let us next discuss how the intensity of the environmental policy k influences the number of equilibria. To this end, we first observe that the marginal profit π' is the difference between the marginal revenue (MR) and the marginal cost (MC). The former does not depend on k, and this fact eases the analysis of the effect of taxation on the existence of multiple equilibria when the MR graph has an increasing trait. For all values of k, the graph of MC is a half-line starting from the value εk and having slope $m + s\varepsilon^2 k$. Figure 3, which is based on a choice of the parameters to be specified later, depicts a typical situation. When $k = 0$ the half-line starts from the origin and crosses the graph of MR just once, for a high value of the output. By increasing k, the half-line representing MC moves upward to the left and it crosses the MR curve at a lower level. This means that when the environmental policy is enforced, marginal costs increase and the output maximizing profits is lower, the equilibrium being still unique. However, there is a threshold $\underline{k}$ for which the half-line is tangent to the MR graph at a lower level than the existing equilibrium. Therefore, when $k > \underline{k}$ two new equilibria emerge. The number of equilibria is three until k reaches a second, higher threshold $\overline{k}$ for which the half-line is again tangent to the MR graph. Then, for all values of k above $\overline{k}$, we come back to a unique equilibrium.[3]

In order to obtain a more general picture, we insert (8) into (10) and write Δ as a cubic function of k, thus getting

$$\Delta(k) = \eta_3 k^3 + \eta_2 k^2 + \eta_1 k + \eta_0$$

where the constants η_i depend on all the other coefficients of the system. It turns out that, under the standing assumption $c^2 < 27bd/8$, both η_3 and η_2 are negative. This fact has two relevant consequences. First, it is always the case that, for all k high enough, $\Delta(k) < 0$, i.e. there is a unique steady state. Second, by Descartes' Rule of Signs, $\Delta(k) = 0$ may have 0, 1 or 2 positive roots, depending on the choice of the coefficients. In the first case, there is a unique steady state for

[2] The boundary case $\Delta = 0$ corresponds to either 2 positive roots, one simple and one double, or a unique triple positive root.

[3] A simular effect can be due to a change in ε ans s. We focus on k because this parameter does not depend on technology applied but on the environmental policy.

all k. In the second case, there are three equilibria when k stays below a given positive threshold and a unique equilibrium otherwise. Finally, in the last case we are in the setting of Figure 3, described above (and the two roots are precisely $\underline{k}$ and $\overline{k}$).

The above description can be seen as a static type result which is an intersection between an exercise of comparative statics and one of multiplicity of equilibria. However, it helps us to highlight two important features of the model. Firstly, from a static point of view, an increase in emission charges leads to a decrease in the quantity produced and, consequently, a decrease in emissions. Secondly, an increase of the parameter k may have a dramatic effect on the existence of multiple equilibria. Therefore, in order to better understand the effects of the environmental policy on emissions, the stability of equilibria may give us further information (see Benhabib et al., 1999 and Menuet et al, 2023). In the next section, we are going to study how the parameter k (but also ε and s) influences the stability of the equilibria.

2.2 Stability of the equilibria

It is well known that a steady state q_* of the dynamics (6), i.e. a positive root of $\pi'(q) = 0$, is asymptotically stable provided $|1 + \gamma\pi''(q_*)| < 1$, that is, provided

$$-\frac{2}{\gamma} < \pi''(q_*) < 0$$

and it is unstable provided one of the two previous inequalities holds (strictly) in the opposite direction. So, we have the following cases for a steady state:

- if $\pi''(q_*) < -2/\gamma$, then q_* locally maximizes the profit, but q_* is unstable
- if $-2/\gamma < \pi''(q_*) < 0$, then q_* locally maximizes the profit and q_* is asymptotically stable
- if $\pi''(q_*) > 0$, then q_* locally minimizes the profit and q_* is unstable

The two boundary cases $\pi''(q_*) = 0$ or $-2/\gamma$ are inconclusive.

From (8) and (9), the conditions for stability of a steady state q_* read

$$-\frac{2}{\gamma} < -(b + m + s\varepsilon^2 k) + 2cq_* - 3dq_*^2 < 0 \tag{11}$$

and we see that they crucially depend on the constant γ (though, they not depend on a).

3 Simulations

In this section we analyse, through simulations, the role of the environmental policy on the number and stability of equilibria. The aim is to highlight relevant elements that, in our view, need to be taken into account in the implementation of the environmental policy. We will focus on the role of k and ε, in particular analyzing the case where the demand function admits up to three steady states. Throughout all simulations we will consider the following simple choice for the parameters d, m, s:

$$d = 1 \qquad m = 1 \qquad s = 1. \tag{12}$$

3.1 The role of k

Let us consider the following set of parameters for the demand function:

$$a = 70 \qquad b = 50 \qquad c = 12.80 \tag{13}$$

They satisfy condition (2), and this guarantees that the graph of the marginal revenue has an increasing trait. The graph of the profit function corresponding to $k = 10$ and $\varepsilon = 0.3$ is reported in Figure 2 on the left: we clearly see the existence of three critical points, two local maxima and a local minimum. This is consistent with Figure 2 on the right, where the graph of the marginal revenue (black) is crossed three times by the half-line of the marginal cost (red).

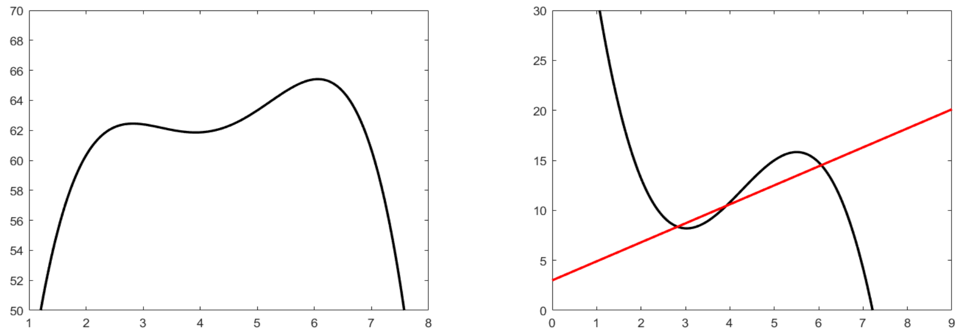


Figure 2: Left: profit vs q; right: Marginal Revenue (black) and Marginal Cost (red) vs q. Both graphs: model as in (12) and (13) with $k = 10$ and $\varepsilon = 0.3$.

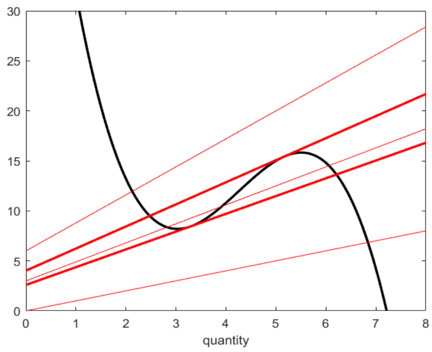
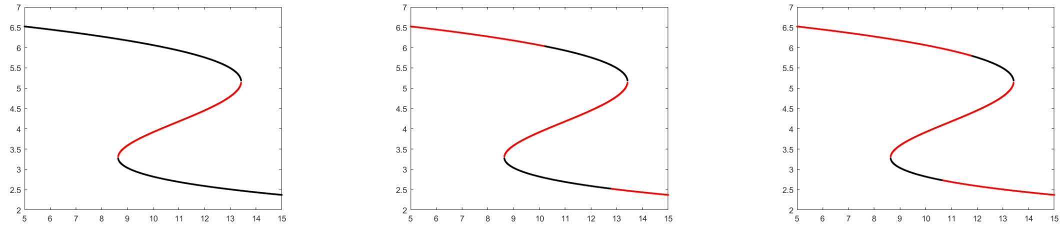


Figure 3: Marginal Revenue (black) and Marginal Cost (red) vs q for model as in (12) and (13) with $\varepsilon = 0.3$ and different values of k (from the bottom to the top half-line: $k = 0$, 8.64, 10, 13.42, 20).

Figure 3 illustrates the critical role of the parameter k in determining the number of steady states. For $k = 0$, a unique steady state exists. As k increases, the marginal costs rise, leading to a decrease in the profit-maximizing quantity, yet the equilibrium remains unique provided k remains below the threshold $\underline{k} \approx 8.64$. Within the interval $k \in (\underline{k}, \overline{k})$, where $\overline{k} \approx 13.42$, three equilibria emerge. For all $k > \overline{k}$, the system reverts to a single equilibrium. Thus, increasing emission charges result in reduced production and, consequently, lower emissions.

3.2 The role of γ on stability

As always in this literature, a crucial role in characterizing stability of equilibria is played by the parameter γ, which describes the responsiveness of the monopolist. Indeed, a steady state may become unstable if the monopolist overreacts to changes in marginal profit. Note, however, that γ plays no role in determining the number and location of equilibria.



(a) $\gamma = 0.15$ (b) $\gamma = 0.30$ (c) $\gamma = 0.45$

Figure 4: Steady states vs k for dynamics (6) for the model as (12) and (13) with $\varepsilon = 0.3$ (black: stable; red: unstable)

In Figure 4 we plot, for three different values of γ, the steady states as a function of k, specifying whether the state is stable (black) or unstable (red). We can observe that the areas of instability increase as γ increases. It is also confirmed that the central steady state, the minimum of the profit function, is always unstable, as discussed above.

3.3 Two equal maximum levels and the role of ε and k

Let us now consider the following set of parameters, still ensuring an increasing trait in the graph of the marginal revenue:

$$a = 70 \qquad b = 52 \qquad c = 13.19 \tag{14}$$

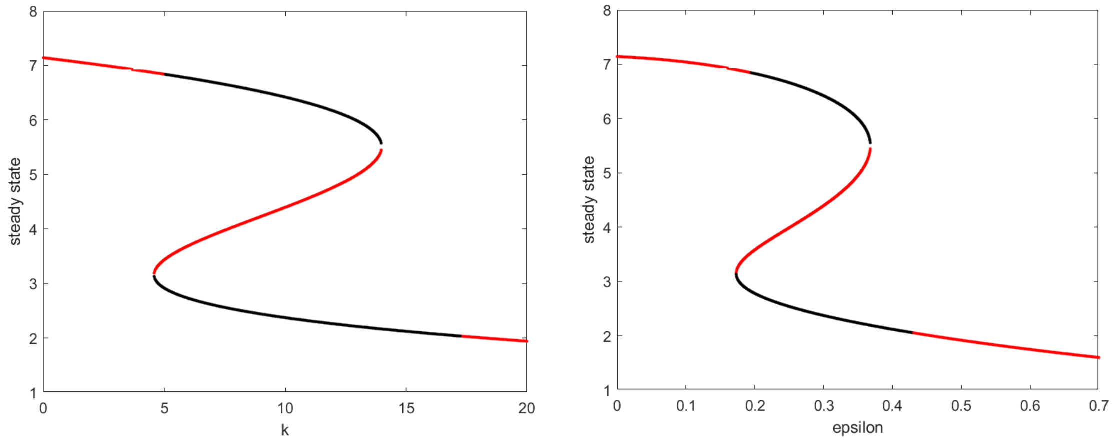
Figure 5 shows the effect that the parameters k and ε have on the number and stability of the equilibria. The left graph confirms the existence of two thresholds $\underline{k}$ and $\overline{k}$ such that three equilibria co-exist whenever $k \in (\underline{k}, \overline{k})$. Remarkably, a similar pattern can be observed when considering the dependence on ε, as it is evident from the right graph. We can also observe that, as both the intensity of environmental policy (k) and the emission per unit of output (ε) increase, the profit-maximising output decreases. From either graphs we can notice that the choice $k = 10$ and $\varepsilon = 0.3$ yields a "symmetric" configuration, in which the two outer equilibria are roughly at a same distance from the inner one. This symmetry is broken as we move apart from those values of the two parameters. Next, sufficiently low or high values of the two parameters make the unique equilibrium unstable. Finally, in both cases, it is possible to choose the parameters (k just below 5 on the left graph, ε just below 0.2 in the right one) under which a stable maximum coexists with an unstable one.

We now fix

$$k = 10 \qquad \varepsilon = 0.3 \qquad \gamma = 0.15 \tag{15}$$

Figure 6 shows that in this case the are two stable local maxima, located at 2.4 and 6.4, respectively, whose profit is roughly at the same level (58.1). We can then easily simulate the dynamics (6), starting from a grids of initial points between $q_0 = 0$ and $q_0 = 9$. The numerical results are not surprising: the long-run average[4] of the quantity coincides with 2.4 when q_0 is below the unstable equilibrium (located at 4.4) and with 6.4 otherwise. As far as the long-run profit is concerned, this is always the common maximum level 58.1, independently from the initial quantity q_0.

[4]The average is computed over 2000 steps of the dynamics, with the first 100 steps considered as "transient", hence discarded.



(a) $\gamma = 0.15$ and $\varepsilon = 0.3$ (b) $\gamma = 0.15$ and $k = 10$

Figure 5: Location of steady states vs k (left) and ε (right) for the model as in (12) and (14). Black: stable; red: unstable.

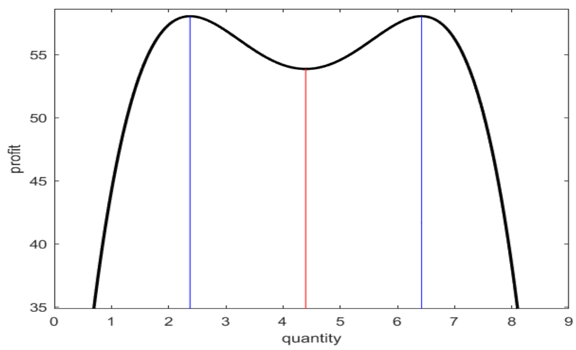


Figure 6: Profit vs quantity for the model as in (12), (14) and (15). Blue: stable equilibrium, red: unstable.

3.4 A lower maximum may lead to higher profits

The most interesting case is when the two local maxima are characterized by a different level of profit. In the following we consider the parameters

$$a = 100 \qquad b = 71.17 \qquad c = 15.42 \tag{16}$$

satisfying condition (2).

Figure 7 (left) shows that the profit function has two maxima located at q_l^* and q_h^*, with $q_l^* < q_h^*$ and $\pi(q_l^*) > \pi(q_h^*)$. Furthermore, the same Figure (right) shows that these two maxima are unstable for values of k below a threshold (roughly equal to 23), while for k roughly in the interval $(23, 47)$, q_l^* is unstable while q_h^* is stable.[5]

We then ran the dynamics (6) for the considered model, with varying initial quantity q_0, and computed the average and the standard deviation of the obtained time series of quantities and profits.[6] Results are displayed in Figure 8 for the case $k = 8$, corresponding to both steady states

[5]This is true also for specific values of ε.

[6]As before, we consider 2000 steps and discard the first 100

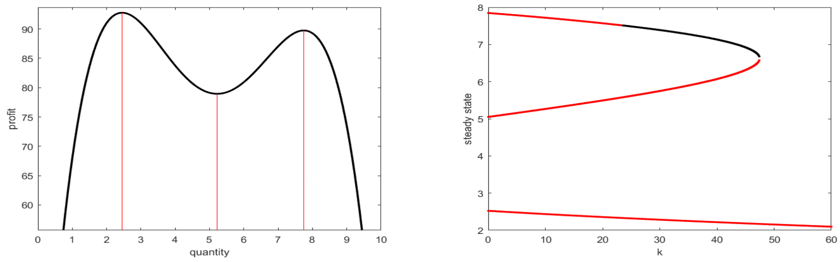


Figure 7: Left: profit vs q for model as in (12) and (16) with $\varepsilon = 0.10$ and $k = 8$; vertical bars: steady states (blue: stable; red: unstable). Right: steady states vs k (black: stable; red: unstable). Both graphs: $\gamma = 0.20$

being unstable. Remarkably, starting from an initial quantity q_0 around q_h^* results in a higher long-run average profit than starting around q_l^*, notwithstanding the fact that $\pi(q_l^*) > \pi(q_h^*)$.

Thus, there is a clear trade-off between policymakers, who would like to reduce the quantity produced, and the monopolist, who would suffer a reduction in profits (on average). At the same time, it is important to stress that this trade-off only emerges through the analysis of equilibrium stability. By comparing the two steady states, it would have been clear (also for the firm) to choose the one with a lower quantity, corresponding to a higher profit (at the steady state).

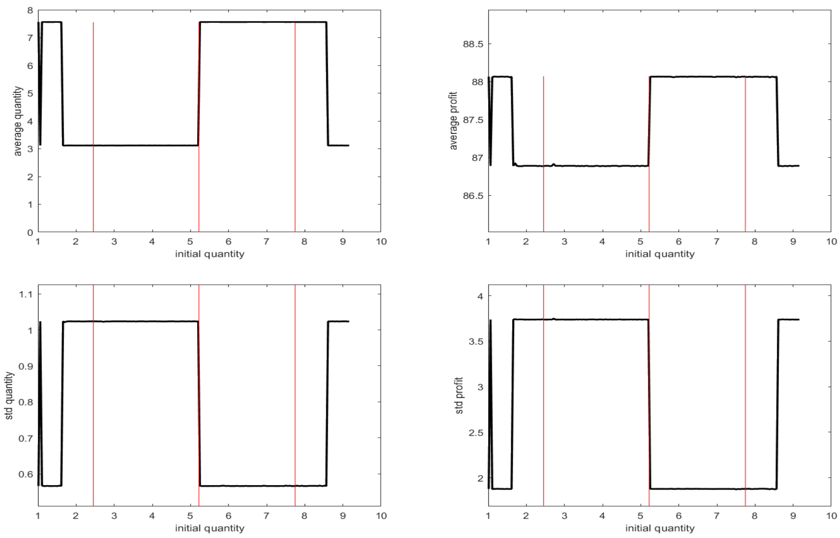
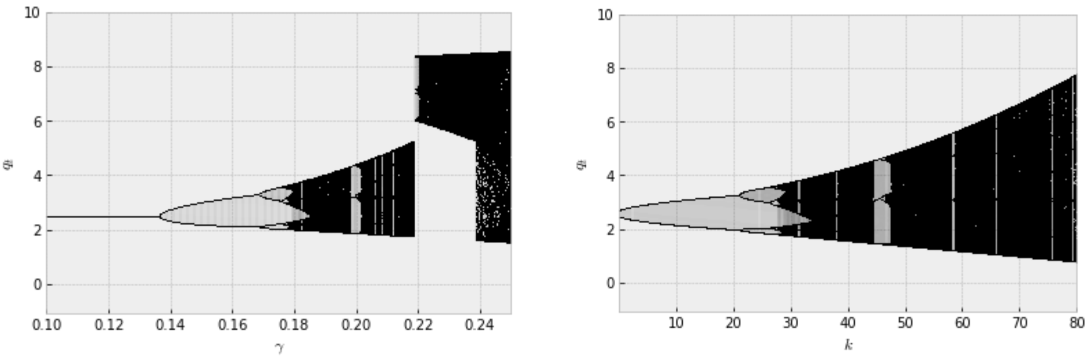


Figure 8: Left: dynamic average (top) and standard deviation (bottom) of a time series of quantities according to dynamics (6) vs initial quantity q_0. Right: same graphs for profit. All graph refer to model as in (12) and (16) with $\varepsilon = 0.10$, $k = 8$ and $\gamma = 0.20$

This result crucially depends on the value of the parameter γ. Indeed, as clearly shown in Figure (9a), an increase in γ, i.e. an higher monopolist's responsiveness, results in an increased production quantity. Moreover, given that q_h^* is stable, this leads to an overall increase in production and emissions.

On the other hand, as Figure (9a) shows, if the reactivity of the monopolist is high ($\gamma \in (0.219, 0.239)$), even though it starts from a low production value, the monopolist falls back to the attractor with the highest production. Moreover, a further increase in the monopolist's reaction leads to the merge of the two attractors and production (as well as emissions) fluctuates between a low and a high value.

Figure (9b) complements this reading by showing that even if γ is set at a *'low'* value, an increase in k - a tightening of environmental policy - leads to an increase in output fluctuations with a consequent reduction in emission control.



(a) $k = 8$ and $\varepsilon = 0.10$

(b) $\gamma = 0.15$ and $\varepsilon = 0.10$

Figure 9: Bifurcation Diagram for γ (left) and k (right). Both graphs refer to dynamics (6) and model as in (12) and (16) and $q_0 = 4$

The policy implication is clear: in addition to emission charges, the Governments' focus should be on the behaviour of firms. Therefore, a study of the dynamics provides insights that go beyond the static analysis.

4 Discussion and Conclusions

The ongoing climate crisis calls for robust economic policies to mitigate the damage, with taxation playing a central role. Our results provide several key insights into the effects of emissions charges.

Increasing emissions charges reduce production and hence emissions. With a bend in the inverse demand function and an increasing segment of the marginal revenue curve, two additional steady states can emerge, leading to the coexistence of two maxima and a minimum. The minimum is always unstable, while the stability of the two maxima depends on cost and demand parameters, as well as the firm's response to marginal returns. The minimum serves as a separating value between two attractors, implying that the effectiveness of emissions policies is highly sensitive to initial conditions. Excessive increases in emissions taxes can destabilise the maxima, leading to unstable production and emissions. Beyond a certain point, the two attractors converge, causing production to fluctuate between a minimum and a maximum and leading to significant emissions volatility.

These findings highlight the complex interplay between emissions charges and market dynamics, and underline the need for carefully calibrated environmental policies to achieve stable and effective outcomes.

References

[1] Alchian, A.A., 1950. Uncertainty, evolutiotion and economic theory. Journal of Political Economy 58, 211-221.

[2] Antoci A., Borghesi S., Iannucci G., Sodini M., 2022, 'Free allocation of emission permits to reduce carbon leakage: an evolutionary approach' in Handbook on trade policy and climate change, vol.76

[3] Bargigli L., Ricchiuti G., 2022, Networks, Heterogeneity and Evolution in Economics: A short Review, Advances in Complex System vol. 25 n.2

[4] Baumol, W.J., Quandt, R.E., 1964. Rules of thumb and optimally imperfect decisions. The American Economic Review 54, 23-46.

[5] Benhabib J., Schmitt-Grohé S., Uribe M., The Perils of Taylor Rules, Journal of Economics Theory 1 June 1999

[6] Bischi G.I, Lamantia F., 2002, 'Nonlinear duopoly games with positive cost externalities due to spillover effects', Chaos, Solitons & Fractals, Volume 13, Issue 4, 701-721

[7] Clower, R.W., 1959. Some theory of an ignorant monopolist. The Economic Journal 69, 705?716.

[8] Costanza R., Wainger L., Folke C., Mäler K-G., 1993. Modeling Complex Ecological Economic Systems: Toward an evolutionary, dynamic understanding of people and nature, BioScience, Volume 43, Issue 8, Pages 545-555, https://doi.org/10.2307/1311949

[9] Formby J. P. , Layson S., Smith W. J., 1982, 'The Law of Demand, Positive Sloping Marginal Revenue, and Multiple Profit Equilibria', Economic Inquiry vol. 2 Issue 2

[10] Irving R.S. 2004. *Integers, polynomials, and rings.* Springer-Verlag New York

[11] IPCC, 2022. 'Climate Change 2022: Mitigation of Climate Change. Working Group III Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change.'

[12] Leonard, D., Nishimura, K., 1999. Nonlinear dynamics in the Cournot model without full information. Annals of Operations Research 89, 165-173.

[13] Matsumoto, A., Szidarovszky, F., 2021. 'Controlling non-point source pollution in Cournot oligopolies with hyperbolic demand' SN Bus Econ 1, 38

[14] Matsumoto, A., Szidarovszky, F., 2022. 'N-firm oligopolies with pollution control and random profits' Asia-Pac J Reg Sci 6, 1017-1039

[15] Menuet, M., Minea, A., Villieu, P., 2023. 'Environmental quality along the process of economic growth: a theoretical reappraisal' Econ Theory https://doi.org/10.1007/s00199-023-01520-1

[16] Naimzada, A., Pireddu, M., 2023. 'Differentiated goods in a dynamic Cournot duopoly with emission charges on output'. Decisions Econ Finan 46, 305-318

[17] Naimzada, A., Pireddu, 2024. 'Comparative dynamics analysis of the environmental policy efficacy in a nonlinear Cournot duopoly with differentiated goods and emission charges', Communications in Nonlinear Science and Numerical Simulation Volume 131, April 2024, 107867

[18] Naimzada A. and Ricchiuti G., 2011. 'Monopoly with Local Knowledge of Demand Function', Economic Modelling, Volume 28, Issues 1-2, Pages 299-307

[19] Naimzada A. and Ricchiuti G., 2008. 'Complex dynamics in a monopoly with a rule of thumb', Applied Mathematics and Computation, Volume 203, Issue 2, Pages 921-925

[20] Negishi, T., 1961. Monopolistic competition and general equilibrium. Review of Economic Studies 27, 136-139

[21] Puu T, 1995. 'The chaotic monopolist' Chaos, Solitons & Fractals, 5 (1) pp. 35-44

[22] Robinson J., 1969. *The Economics of Imperfect Competition*, second edition, ST. Martin's Press

[23] Silvestre, J., 1977. A model of general equilibrium with monopolistic behavior. Journal of Economic Theory 16, 425-442.

[24] Tuinstra, J., 2004. A price adjustment process in a model of monopolistic competition. International Game Theory Review 6, 417-442.

[25] Walters A., 1980. 'A note on monopoly equilibrium', Economic Journal, vol. 90