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Environmental Responsibility as a Strategic Driver for Insurance Decisions

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Abstract

The increase in environmental accidents encourages companies to take out insurance against potential losses. In addition, firms may also choose to engage in environmentally responsible (ECSR) activities in the form of voluntary emission reductions to mitigate the probability of environmental losses. We study this problem in an industry consisting of a risk-averse firm in which environmental accidents can occur. In this industry, consumers are environmentally concerned, a tax on emissions is in place, and the firm chooses its type (profit-seeking or ECSR), its demand for insurance against environmental accidents, the emission reduction investment, and the production level. Our results show that engaging in ECSR activities may not only be a response to environmental policy or consumer demands, but also a response to a tight insurance market with high premia.

Keywords: Demand for insurance, endogenous loss, environmental loss.
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1 Introduction

The intensification of environmental accidents can impose significant financial burdens on firms, prompting them to seek protection against such risks. Corporate environmental insurance refers to insurance policies designed to protect firms against financial losses arising from environmental liabilities, such as the costs of remediation, litigation and compensation for environmental incidents (see Sjoquist and Sjoquist, 1992, Swafford and Dallman, 1999, and more recently Freedman et al., 2007).

Another (not necessarily alternative) strategy is for the firm to reduce its environmental impact once it has established that production-related emissions have a positive impact on the size and likelihood of an environmental disaster occurring. In practice, a firm may choose to formally embrace environmental corporate social responsibility (ECSR) to voluntarily reduce its environmental impact in order to mitigate expected losses from negative environmental events.

Classical insurance theory studies the demand for insurance and self-insurance as substitutes (see Ehrlich and Becker, 1972, Briys and Schlesinger, 1990, Courbage, 2001 and Pannequin et al., 2020). In a situation of environmental accident risk, self-insurance can be interpreted as investment in green innovation or emission reduction technology.

Empirical evidence shows that the use of environmental insurance is related to environmental performance and green innovation (Chen et al., 2022, Wu et al., 2022, Ning et al., 2023, Zhu et al., 2023 Shi et al., 2023, and Wen et al., 2024). Moreover, in a recent paper, Kong et al. (2020) find that ECSR activities can significantly reduce firms' idiosyncratic risk, which is positively related to the risk of environmental accidents. However, the relationship between the occurrence of environmental accidents and the adoption of ECSR practices has not been directly investigated, either empirically or theoretically.

In this paper, we analyse the relationship between the adoption of ECSR practices and the demand for insurance against environmental accidents. We consider a single firm industry in which consumers are environmentally concerned, production is polluting and there is a tax on emissions. In addition, we assume the existence of environmental accidents, which are partly due to the level of emissions, and which can cause a financial loss to the firm. In this context, ECSR activities involve voluntary emission reductions and are therefore strategic not only in terms of consumer preferences and environmental policy, but also in terms of expected financial losses from environmental accidents. In addition, ECSR practices become a substitute for the insurance market by influencing the premiums offered and the opportunities in this sector.

The management of the firm is organised into three bodies (Board of Directors, Risk Management Department and Operations Department), each with different objectives. The Board of Directors decides whether to adopt an ECSR statute. In practice, this amounts to a voluntary reduction in production emissions, regardless of the existence of a policy. Even in the absence of negative natural events, the firm may have an incentive to adopt ECSR activities because consumers are environmentally concerned and would appreciate (in terms of willingness to buy) the firm's efforts to reduce its impact. The Risk Management Department chooses the level of coverage against environmental accidents. Its decision is based on expected profits and the cost of the premium. Finally, the Operations Department determines the production strategy and the choice of investment in emission reduction technology.

When analysing the decisions of the Operations Department, we find that, intuitively, a higher level of production induces a higher level of insurance coverage, while a higher level of investment in emission reduction is associated with a lower level of insurance coverage.

In the analysis of Risk Management decisions, we examine the role of market size, tax on emissions, the impact of emissions on environmental accidents and the loading factor. Irrespective of the type of firm, a larger market size stimulates the level of coverage, while the level of tax, the impact of emissions on damage and the loading factor reduce the demand for insurance. When the firm is of the ECSR type, we find that insurance coverage increases with environmental concern and decreases with the level of voluntary internalisation of emissions.

The Board of Directors' decision to adopt an ECSR statute is the focus of this paper. It shows that as the loading factor increases, so does the incentive to convert the firm into an ECSR. Note that the size of the loading factor reflects the characteristics of the insurance market in terms of cost efficiency and the ability of insurers to charge a premium. Since, in practice, a higher loading factor translates into a higher equilibrium premium, it follows that the adoption of ECSR activities somewhat mitigates the effect of higher premiums in the environmental insurance market.

The remainder of the paper is organised as follows. Section 2 briefly outlines the relevant literature and Section 3 introduces the model. Section 4 presents the results, considering first a standard firm, then a firm that adopts ECSR activities, and finally the endogenous choice of the ECSR adoption. Section 5 concludes. All proofs can be found in the Appendix.

2 Literature

The paper is related to the literature on insurance and the literature on environmental and corporate social responsibility.

With regard to the former, our contribution fits into three strands of literature: that on insurance demand, that on self-insurance, and that on insurance against environmental accidents.

Smith (1968) is the first paper to focus on the demand for insurance, studying the optimal level of insurance coverage.[1] Mayers and Smith (1982) provides further motivations for why firms demand insurance: risk shifting, tax benefits and cost reduction. In addition to these, MacMinn and Witt (1987), Schnabel and Roumi (1989) and MacMinn and Han (1990) have added the cost of financial distress, and Han (1996) has emphasised the role of managerial compensation. The strategic analysis of insurance demand was introduced by Ashby and Diacon (1998) in a simple Cournot-Nash duopoly and then extended by Seog (2006). Seog (2006) assumes that the financial loss is composed of two parts, one exogenous (random) and one endogenous (deterministic), the latter being related to the output produced by the firms that also choose the insurance coverage rate. Of this strand of literature, our paper is mainly related to Seog (2006). Our modelling of loss is similar, but the endogenous component depends on the level of emissions rather than on output, because, unlike Seog (2006), our focus is on losses resulting from

[1] An exhaustive tractation of the demand for insurance is given in Szpiro (1988).

environmental accidents rather than on losses in general.

The paper also relates to the literature on self-insurance, which can be defined as an action that an agent can do to reduce the intensity of loss. The seminal work here is Ehrlich and Becker (1972), who examines the relationship between insurance demand and self-insurance in the form of behaviours that reduce the probability of hazardous events. Developments in this area include Briys and Schlesinger (1990), Courbage (2001) and, more recently, Pannequin et al. (2020). A common result in this literature is the substitution effect between the demand for insurance and self-insurance. In our analysis, self-insurance amounts to the reduction of polluting emissions that contribute to the cause of the environmental accident. In addition, ECSR activities correspond to voluntary (i.e. not driven by environmental policies) emission reduction, which can be interpreted as another way of self-insurance.

The third strand of insurance literature related to our paper is the study of environmental accidents insurance. This body of research has largely focused on how households choose to purchase property in areas prone to natural disasters and the resulting impact on property values (Bin and Landry, 2013, Georgic and Klaiber, 2022). It also examines the socio-economic characteristics of households relocating to high-risk areas (Bakkensen and Ma, 2020), their willingness to pay for insurance (Botzen et al., 2009 and Bradt et al., 2021, among others), the distribution of total losses (Conte and Kelly, 2018) and the role played by insurance transparency in the long-run abatement decisions (Müller-Fürstenberger and Schumacher, 2015). In contrast to this literature, the present paper focuses on the analysis of corporate rather than household environmental insurance.

Finally, the paper is linked to the literature on strategic environmental corporate social responsibility. Generally speaking, *Environmental Corporate Social Responsibility* (ECSR) is defined as a firm's formal efforts to minimise its negative impact on the natural environment through sustainable practices and pollution reduction. The focus on "strategic" refers to the fact that the adoption of these practices is assumed to be beneficial to the firm. Examples could be better financial performance (Wu and Shen, 2013), lower cost of capital (El Ghoul et al., 2011) or lower crash risk (Kim et al., 2014).

The integration of environmental concerns into corporate social responsibility

(CSR) has evolved over time to the point where the relevance of environmental concerns *per se*, together with social responsibility, has been recognised. As a result, the literature on ECSR has emerged from the extensive research on strategic CSR (Baron, 2001) which, for the sake of brevity, will be omitted from this section of the literature.

A commonly accepted way of modelling ECSR behaviour is to acknowledge the existence of environmentally concerned consumers. Papers using this approach include Manasakis et al. (2013), Manasakis et al. (2014), Liu et al. (2015), Xu and Lee (2023), Fang and Zhao (2023), Kouider Aissa and Tampieri (2024) and Clò et al. (2024). This type of consumer places a higher value on goods produced with a low environmental impact and is therefore willing to pay a higher price for such products. In turn, a firm may have a strategic incentive to engage in ECSR activities in the form of emission reductions in order to attract these consumers. Under certain conditions, ECSR firms produce more and earn higher profits than firms that focus only on profits. Here, the practice of reducing emissions further stimulates demand, so that the cost of environmental responsibility is offset by green consumers paying more for ECSR products than for standard goods. In line with this approach to modelling ECSR, the present analysis refers to this literature as the firm's Board of Directors decides whether or not to adopt ECSR practices in the form of emission reductions with the aim of maximising the firm's profits.

3 The model

Consider a single-product, single-firm industry. In its own decision process (described in detail below), the firm decides whether to pursue profit maximisation or to adopt an environmentally responsible statute by internalising some of its own emissions. We denote a standard profit-seeking (PS) firm by P and an environmentally socially responsible (ECSR) firm by E.

3.1 Demand side

We assume that consumers are "environmentally aware": if the firm becomes an ECSR, consumers recognise its commitment to reducing production emissions because they are willing to pay a higher price for a good produced by it.

A representative consumer's utility depends on the type choice of the firm $i \in \{P, E\}$, and it is given by:

$$U_i = \left(a + \mathbb{1}_{\{i=E\}}\theta\eta\right) q_i + \frac{1}{2}q_i^2 + c_0. \tag{1}$$

In Eq. (1), q represents the quantity of the good purchased by firm i, while a represents the component of willingness to pay common to PS and ECSR products. Finally, $\theta\eta$ is the additional willingness to pay for the good if the firm becomes an ECSR. This is given by the degree of environmental awareness of consumers (denoted by $\theta \in (0, 1]$) and the degree of emissions internalisation of ECSR firms (denoted by $\eta \in (0, 1]$). The indicator function $\mathbb{1}_{\{i=E\}}$ clarifies that it is only active if the firm chooses the E type and zero otherwise, i.e., if the firm is PS.

The consumer's budget constraint is given by

$$c_0 + p_i q_i = Y, \tag{2}$$

where c_0 represents the composite good, whose price is normalised to 1 and used as the *numeraire*, while Y denotes income. Maximising Eq. (1) with respect to q_i and subject to Eq. (2), yields the following inverse demand function:

$$p_i = a + \mathbb{1}_{\{i=E\}}\theta\eta - q_i. \tag{3}$$

3.2 The firm's governance

Within the firm, different managers perform different functions. In particular, the firm is made up of three bodies: the *Board of Directors*, denoted by the subscript b, the *Risk Management Department*, denoted by the subscript r, and the *Operations Department*, denoted by the subscript o. All these bodies play a role in the management of the firm, with different objectives. We assume that all bodies have constant absolute risk aversion. Their objective function is given by

$$O_{ji} = -\exp(-Av_{ji}), \tag{4}$$

with $j \in \{b, r, o\}$, A is the coefficient of risk aversion and v_{ij} is a function that depends on the department and the firm's type considered.

3.2.1 Operations Department

The Operations Department manages the day-to-day operations of production and investment. Production has constant marginal costs c. Moreover, production is polluting and there is an environmental policy in place: the Environmental Agency sets a tax rate τ on the level of emissions. Hence, the Operations Department has an incentive in investing in emission reduction technology to curb the fiscal burden. We assume that emissions of firm i, denoted as e_i are given by the level of production q_i minus the investment in abatement, denoted by z_i, so that

$$e_i = q_i - z_i.$$

Investment costs are convex (quadratic) and independent of the quantity produced.

This type of emission reduction technology is called *end-of-pipe*, which refers to pollution control methods that treat pollutants at the point of discharge, rather than preventing their formation during the production process (Dupuy, 1997). Typically, this approach mitigates environmental impacts by implementing treatment systems at the final stage before pollutants are released into the environment.

It follows that a firm's operating profits are

$$R_i = (p_i - c)q_i - \frac{z_i^2}{2} - \tau e_i. \tag{5}$$

An environmental accident causes a financial loss to the firm. This loss is given by two components: first, an exogenous component, depending on the idiosyncratic level of the environmental accident, which is given by a random variable $\widetilde{w}$ normally distributed with mean μ and variance σ^2. Second, an endogenous component, depending on the level of emissions due to the firm's production. In other words, the higher the emission reduction, the lower the loss due to the environmental accident, so that emission reduction can act as a "self insurance" mechanism.

We are now in a position to define the firm's *profits*: these are operating profits minus the financial loss due to the environmental disaster for the uncovered share:

$$\pi_i = R_i - (1 - \beta_i)\widetilde{w}\phi e_i. \tag{6}$$

In the financial loss $(1 - \beta_i)\widetilde{w}\phi e_i$, $(1 - \beta_i)$ is the retention rate and $\phi > 0$ is a parameter representing the impact of emissions on the financial loss.

The objective of the Operations Department is set by the Board of Directors, which, as explained in more detail below, decides whether to adopt an ECSR statute. In practice, this requires the internalisation of own emissions, despite the existence of an environmental policy. In particular, we assume that the ECSR firm reduces its emissions by a fraction η. While this internalisation represents an additional cost for the firm, it could be compensated by the higher valuation by consumers, who are willing to pay more for the firm's product if it has been produced with a lower environmental impact.

It follows that the Operations Department's objective function is given by $O_{oi}(v_{oi})$, where

$$v_{oi} = \pi_i - \mathbb{1}_{\{i=E\}}\eta e_i. \tag{7}$$

By Eq. (7), given that the exogenous component of the environmental loss is normally distributed, and denoting $\alpha = \frac{A}{2}$, the Operations Department's expected objective is[2]

$$Ev_{oi} = R_i - (1 - \beta_i)(\mu + \alpha\sigma)\phi e_i - \mathbb{1}_{\{i=E\}}\eta e_i. \tag{8}$$

3.2.2 Risk Management Department

The firm may protect itself from the risk of the financial loss by buying an insurance, which may partially or fully cover the loss, based on the percentage the firm chooses to cover itself. This task is the role of the Risk Management Department. In particular, this department chooses an insurance coverage $\beta_i \in [0, 1]$, where $\beta_i = 0$ means no insurance and $\beta_i = 1$ means full insurance.

The insurance market is competitive, so the insurance premium is given by the

[2]For notational reasons, we use the volatility instead of the variance.

expected value principle:

$$P_i = (1+\lambda)\mu\phi e_i\beta_i, \tag{9}$$

where $\lambda \geqslant 0$ is the loading factor. The Risk Management Department establishes its coverage level, β_i, by maximising the firm's objectve, established by the Board of Directors, minus the cost of the premium (Seog, 2006). Thus its objective function is given by $O_{ri}(v_{ri})$, where

$$v_{ri} = \pi_i - \mathbb{1}_{\{i=E\}}\eta e_i - P_i. \tag{10}$$

Hence, the Risk Management's expected objective is

$$Ev_{ri} = R_i - (1-\beta_i)(\mu+\alpha\sigma)\phi e_i - \mathbb{1}_{\{i=E\}}\eta e_i - (1+\lambda)\mu\phi e_i\beta_i. \tag{11}$$

Note that optimal insurance coverage is usually interpreted as the demand for insurance.

3.2.3 Board of Directors

As anticipated above, the Board of Directors is the body that establishes the firm's objective, either PS or ECSR. The Board's choice is based on expected profits: the statute that ensures the highest expected profits is chosen. The board's objective is therefore given by $O_{bi}(v_{bi})$, where $v_{bi} = \pi_i$. Thus, its expected objective function is

$$Ev_{bi} = R_i - (1-\beta_i)(\mu+\alpha\sigma)\phi e_i. \tag{12}$$

3.3 Timing

The sequence of the decisions within the firm is as follows. First, the Board of Directors evaluates whether an ECSR statute should be adopted. In the second stage, the Risk Management Department chooses the degree of insurance coverage. In the third stage, the Operations Department chooses quantities and investment in emission reduction technology. The solution concept is subgame perfection by backward induction, with the sequence of stages ensuring time consistency.

4 Results

In this section we present the results of the paper. The presentation is as follows. First, we derive the industry equilibrium separately, according to the Board of Directors' choice in the first stage. We then focus on the Board's choice of type and, more generally, on a comparison between the two industries. For simplicity, we define $\gamma = a - c$, which represents the size of the market.

4.1 Profit seeking type

4.1.1 Operations Department's choice

Consider first the standard, profit maximising firm. In the third stage, the Operations Department maximises profits with respect to quantity q_P and emission reduction investment z_P, so that its problem is

$$\max_{q_P, z_P} Ev_{oP} = R_P - (1 - \beta_P)(\mu + \alpha\sigma)\phi e_P, \tag{13}$$

$$\text{s.t. } q_P \geqslant 0, z_P \geqslant 0, q_P - z_P \geqslant 0. \tag{14}$$

The first order conditions, for interior solutions, with respect q_P and z_P are:[3]

$$\frac{\partial \pi_P}{\partial q_P} = \gamma - 2q_P - \tau - (1 - \beta_P)(\mu + \alpha\sigma)\phi = 0, \tag{15}$$

$$\frac{\partial \pi_P}{\partial z_P} = -z_P + \tau + (1 - \beta_P)(\mu + \alpha\sigma)\phi = 0. \tag{16}$$

Solving Eq. (15) with respect to q_P and z_P, we obtain the optimal values as summarised in the next proposition.

Proposition 1. *The equilibrium level of quantities and investment in emission reduction technology of the PS Operations Department are*

$$q_P^* = \frac{1}{2}[\gamma - \tau - (1 - \beta_P)(\mu + \alpha\sigma)\phi], \tag{17}$$

$$z_P^* = \tau + (1 - \beta_P)(\mu + \alpha\sigma)\phi. \tag{18}$$

[3] See the Appendix for a complete derivation of all solutions. We exlude boundary solutions because if $q_P = z_P$, then the insurance market disappears.

Note that the results in Proposition 1 are positive and ensure a positive investment in insurance coverage whenever $\gamma > \widehat{\gamma}_P$, where

$$\widehat{\gamma}_P \equiv 3[\tau + (1-\beta_P)(\mu + \alpha\sigma)\phi].$$

Therefore, in the following we assume

Assumption 1. *Let* $\gamma > \widehat{\gamma}_P$.

Proposition 1 shows that there is moral hazard, i.e. an increase in the level of coverage stimulates the level of production and restrains the level of investment, implying a higher level of emissions in equilibrium. This result is obtained by differentiating the equilibrium values with respect to the coverage level:

$$\frac{\partial q_P^*}{\partial \beta_P} = \frac{1}{2}(\mu + \alpha\sigma)\phi > 0,$$
$$\frac{\partial z_P^*}{\partial \beta_P} = -(\mu + \alpha\sigma)\phi < 0,$$

so that one may summarise

Corollary 1. *Let Assumption 1 hold. In the PS firm, an increase in insurance coverage leads to an increase in output and a decrease in abatement investment.*

The results in Corollary 1 are natural in the general literature of insurance and self-insurance, dating back to Ehrlich and Becker (1972).

4.1.2 Risk Management Department's choice

In the second stage, the Risk Management Department establishes its coverage level, β_P by maximising its expected objective function from Eq. (11), that is,

$$\max_{\beta_P \in [0,1]} Ev_{rP} = R_P^* - (1-\beta_P)(\mu + \alpha\sigma)\phi e_P^* - (1+\lambda)\mu\phi e_P^* \beta_P.$$

The first order condition is

$$\frac{\partial E v_{rP}}{\partial \beta_P} = \gamma \frac{\partial q_P^*}{\partial \beta_P} - 2\frac{\partial q_P^*}{\partial \beta_P} q_P^* - \left[\left(\frac{\partial q_P^*}{\partial \beta_P} - \frac{\partial z_P^*}{\partial \beta_P}\right)(1-\beta_P) - (q_P^* - z_P^*)\right](\mu + \alpha\sigma)\phi$$
$$- \left[\left(\frac{\partial q_P^*}{\partial \beta_P} - \frac{\partial z_P^*}{\partial \beta_P}\right)(1-\beta_P) + (q_P^* - z_P^*)\right](1+\lambda)\phi\mu = 0. \tag{19}$$

Note that the second order condition is verified as long as

$$\frac{\partial^2 Ev_{rP}}{\partial \beta_P^2} = -\frac{3}{2}(\alpha\sigma - \lambda\mu) < 0,$$

which is verified for $\lambda < \overline{\lambda}$, where

$$\overline{\lambda} \equiv \frac{\alpha\sigma}{\mu}.$$

Solving Eq. (19) with respect to β_P, we are able to determine the equilibrium coverage level when the firm is PS. Define

$$\underline{\lambda}_P = \frac{[\gamma - 3\tau - 3(\mu + \alpha\sigma)\phi]\alpha\sigma - 3(\mu^2 - \alpha^2\sigma^2)\phi}{[\gamma - 3\tau + 3(\mu + \alpha\sigma)\phi]\mu},$$

where

$$\overline{\lambda} - \underline{\lambda}_P = \frac{3\left(\mu + \alpha\sigma\right)^2 \phi}{\left(\gamma - 3\tau + 3\mu\phi + 3\alpha\sigma\phi\right)\mu} > 0,$$

for $\gamma > \widehat{\gamma}_P$. The following proposition summarises the Risk Management's choice when the firm is of PS type:

Proposition 2. *Let Assumption 1 hold. The equilibrium coverage level of the PS Risk Management Department is*

$$\beta_P^* = \begin{cases} 1 & \text{if } \lambda \leqslant \underline{\lambda}_P, \\ \dfrac{[\gamma - 3\tau - 3(\mu + \alpha\sigma)\phi](\alpha\sigma - \lambda\mu)}{3(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi}, & \text{if } \lambda \in (\underline{\lambda}_P, \overline{\lambda}). \\ 0 & \text{if } \lambda \geqslant \overline{\lambda}. \end{cases}$$

From Proposition 2, we may evaluate how the coverage choice changes according to parameter values.

Corollary 2. *The demand for insurance of a PS firm increases with the market size, and decreases with the tax on emissions, with the impact of emissions on the financial loss and with the loading factor.*

4.1.3 Equilibrium profits

Given Proposition 1 and Proposition 2, we can write equilibrium profits as:

$$\pi_P^* = (q_P^*)^2 - \frac{(z_P^*)^2}{2} + \tau z_P^* + (1-\beta_P^*)(\mu + \alpha\sigma)\phi z_P^*.$$

The following proposition illustrates how changes in the tax on emissions, the impact of emissions on the financial loss and the loading factor

Proposition 3. *Equilibrium profits of a PS firm are decreasing in the tax on emissions, the impact of emissions on the financial loss and the loading factor.*

As might be expected, profits fall as tax and insurance costs rise and as the severity of the loss increases. This means that taxes and insurance have a greater impact on operating profits than on losses (with the opposite sign). In a sense, operating profits carry more weight in total profits.

4.2 ECSR type

In this section, we evaluate the alternative case in which the firm adopts ECSR practices.

4.2.1 Operations Department's choice

In the third stage, the Operations Department maximises Eq. (8), given by profits minus the internalisation of emissions, with respect to quantity q_E and emission reduction investment z_E, i.e.

$$\max_{q_P, z_P} Ev_{oP} = R_E - (1-\beta_E)(\mu + \alpha\sigma)\phi e_E - \eta e_E, \tag{20}$$

$$\text{s.t. } q_E \geqslant 0, z_E \geqslant 0, q_E - z_E \geqslant 0. \tag{21}$$

From Eq. (20), the first order conditions for interior solutions, with respect q_E and z_E are:[4]

$$\frac{\partial O_E}{\partial q_E} = \gamma + \theta\eta - 2q_E - \tau - \eta - (1-\beta_E)(\mu + \alpha\sigma)\phi = 0, \tag{22}$$

$$\frac{\partial O_E}{\partial z_E} = -z_E + \tau + \eta + (1-\beta_E)(\mu + \alpha\sigma)\phi = 0. \tag{23}$$

Solving Eq. (22) with respect to q_E and z_E, respectively, we obtain the optimal values as summarised in the next proposition.

Proposition 4. *In the third stage, the equilibrium level of quantities and investment in emission reduction technology of the ECSR firm are*

$$q_E^* = \frac{1}{2}[\gamma - \tau - (1-\beta)(\mu + \alpha\sigma)\phi], \tag{24}$$

$$z_E^* = \tau + (1-\beta)(\mu + \alpha\sigma)\phi. \tag{25}$$

Like in Proposition 1, we are focusing on the equilibrium where insurance coverage can be a relevant option. This occurs whenever $\gamma > \widehat{\gamma}_E$, where

$$\widehat{\gamma}_E = (3-\theta)\eta + 3[\tau + (\mu + \alpha\sigma)\phi].$$

In what follows, we assume

Assumption 2. *Let* $\gamma > \widehat{\gamma}_E$.

A quick glance to Proposition 4 shows that, like in the PS case, moral hazard occurs. Hence the results in Corollary 1 hold also when the firm is ECSR, so that

$$\frac{\partial q_E^*}{\partial \beta_E} = \frac{(\mu + \alpha\sigma)\phi}{2},$$

$$\frac{\partial z_E^*}{\partial \beta_E} = -(\mu + \alpha\sigma)\phi.$$

[4] Like in the previous case, in the Appendix we provide a complete derivation of all solutions.

4.2.2 Risk Management Department's choice

In the second stage the Risk Management Department chooses its coverage level, β_E, to maximise its expected objective in Eq. (11). Hence, in the second stage, the ECSR Risk Management Department's problem is:

$$\max_{\beta_E \in [0,1]} Ev_{rE} = R_E^* - (1-\beta_E)(\mu+\alpha\sigma)\phi e_E^* - \eta e_E^* - (1+\lambda)\mu\phi e_E^* \beta_E. \tag{26}$$

The first order condition of Eq. (26) is

$$\begin{aligned}\frac{\partial Ev_{rE}}{\partial \beta_E} &= (\gamma+\theta\eta)\frac{\partial q_E^*}{\partial \beta_E} - 2\frac{\partial q_E^*}{\partial \beta_E}q_E^* - \left(\frac{\partial q_E^*}{\partial \beta_E} - \frac{\partial z_E^*}{\partial \beta_E}\right)\eta \\ &- \left[\left(\frac{\partial q_E^*}{\partial \beta_E} - \frac{\partial z_E^*}{\partial \beta_E}\right)(1-\beta_E) - (q_E^* - z_E^*)\right](\mu+\alpha\sigma)\phi \\ &- \left[\left(\frac{\partial q_E^*}{\partial \beta_E} - \frac{\partial z_E^*}{\partial \beta_E}\right)(1-\beta_E) + (q_E^* - z_E^*)\right](1+\lambda)\phi\mu = 0.\end{aligned} \tag{27}$$

The second order condition is verified whenever

$$\frac{\partial^2 Ev_{rE}}{\partial \beta_E^2} = -\frac{3}{2}(\mu+\alpha\sigma)(\mu-\alpha\sigma+2\lambda\mu)\phi^2 < 0,$$

which occurs for

$$\lambda > \hat{\lambda} \equiv \frac{\alpha\sigma-\mu}{2\mu}.$$

Again, solving Eq. (27) with respect to β_E, we are able to determine the equilibrium coverage level when the firm is ECSR. First, define

$$\underline{\lambda}_E = \frac{[\gamma-(3-\theta)\eta-3\tau-3(\mu+\alpha\sigma)\phi]\alpha\sigma - 3(\mu^2-\alpha^2\sigma^2)\phi}{[\gamma-(3-\theta)\eta-3\tau+3(\mu+\alpha\sigma)\phi]\mu},$$

where

$$\bar{\lambda} - \underline{\lambda}_E = \frac{3\left(\mu+\alpha\sigma\right)^2\phi}{\left(\gamma-3\tau-3\eta+\theta\eta+3\mu\phi+3\alpha\sigma\phi\right)\mu} > 0,$$

and

$$\underline{\lambda}_E - \hat{\lambda} = \frac{\left(\gamma-3\tau-3\eta+\theta\eta+3\mu\phi-3\alpha\sigma\phi\right)\left(\mu+\alpha\sigma\right)}{2\left(\gamma-3\tau-3\eta+\theta\eta+3\mu\phi+3\alpha\sigma\phi\right)\mu} > 0,$$

for $\gamma > \hat{\gamma}_E$.

Proposition 5. *Let Assumption 2 hold. The equilibrium coverage level of the ECSR Risk Management Department is*

$$\beta_E^* = \begin{cases} 1 & \text{if } \lambda \leqslant \underline{\lambda}_E, \\ \dfrac{[\gamma - (3-\theta)\eta - 3\tau - 3(\mu+\alpha\sigma)\phi](\alpha\sigma - \lambda\mu)}{3(\mu+\alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi} & \text{if } \lambda \in (\underline{\lambda}_E, \overline{\lambda}), \\ 0 & \text{if } \lambda \geqslant \overline{\lambda}. \end{cases}$$

Note $\lambda < \overline{\lambda}$ implies that $\alpha\sigma - \lambda\mu > 0$, namely the insurance coverage is positive if the reduction in the cost of risk is greater than the adjusted loading factor.

From Proposition 5, we are able to analyse the equilibrium changes in the coverage choice according to variations in the parameter values. The following corollary mirrors Corollary 2 when the firm is ECSR.

Corollary 3. *The demand for insurance of a ECSR firm increases with the market size, and decreases with the tax on emissions, with the impact of emissions on the financial loss and with the adjusted loading factor.*

By Corollary 3, the comparative statics results are qualitatively similar to those of the PS firm. The difference regards the impact of emissions over the financial loss. Indeed, we may compare:

$$\frac{\partial \beta_P^*}{\partial \phi} - \frac{\partial \beta_E^*}{\partial \phi} = \frac{(1-\theta)\,(\lambda\mu - \alpha\sigma)\,\eta}{3\,(\mu - \alpha\sigma + 2\lambda\mu)\,(\mu + \alpha\sigma)\,\phi^2} > 0.$$

An increase in the impact of emissions over the loss due to the environmental disaster, the PS Risk Management lowers its coverage faster than the ECSR Risk Management.

If the firm is ECSR, we can also evaluate variations in consumer environmental awareness θ and the firm's commitment to reducing emissions η.

Corollary 4. *The ECSR's demand for insurance increases with the consumers' environmental awareness and decreases with the ECSR's degree of emission internalisation.*

The first part of Corollary 4 is intuitive. An increase in θ means an increase in consumer demand, which in turn allows Risk Management to increase the level of coverage. The second part of Corollary 4 is more interesting: it shows that an increase in the internalisation of emissions reduces the coverage level. This result shows the fact that voluntary emission reduction, as established by the ECSR statute, acts as a self-insurance mechanism, which is naturally a substitute of the insurance coverage.

4.2.3 Equilibrium profits

Given Proposition 4 and Proposition 5, the equilibrium profits of the ECSR firm may be written as:

$$\pi_E^* = (q_E^*)^2 + \eta q_E^* - \frac{(z_E^*)^2}{2} + \tau z_E^* + (1-\beta_E)(\mu + \alpha\sigma)\phi z_E^*.$$

The following proposition illustrates how changes in the tax on emissions, the impact of emissions on the financial loss and the loading factor

Proposition 6. *Equilibrium profits of the ECSR firm are decreasing in the tax on emissions, the impact of emissions on the financial loss and the loading factor.*

The results of Proposition 6 mirror those of Proposition 3. By evaluating the impact on profits of consumers' environmental concern and ECSR's emissions internalisation, we obtain

Proposition 7. *Equilibrium profits of an ECSR firm are increasing in consumers' environmental concern and decreasing in the level of emission internalisation.*

By Proposition 7, an increase in θ leads to higher operating profits, although it also increases environmental damage. Conversely, an increase in η is more ambiguous, as it has a positive effect on demand and reduces environmental damage, but represents a cost to the ECSR firm.

4.3 The firm's type choice

This section focuses on the Board of Directors problem. First, we compare the equilibrium elements in the two types of firms. We then analyse the conditions under which the Board chooses one type over the other. Again, we will focus on the situation where, for both types of firm, covering the financial loss is appropriate according to the objective of the Risk Management Department. We will therefore impose

Assumption 3. *Let* $\lambda \in \left(\widehat{\lambda}, \overline{\lambda}\right)$.

Begin with the comparison of equilibria.

Proposition 8. *Let Assumption 3 hold. In equilibrium, the level of production, emissions and insurance coverage is higher and the level of investment is lower when the firm is of PS type.*

We then consider the Board of Directors' decision on whether or not to adopt an ECSR statute. The Board's problem is given by the difference between its expected objective function without and with the adoption of ECSR practices:

$$\begin{aligned} Ev_{bP} - Ev_{bE} = R_P^* - (1 - \beta_P^*)(\mu + \alpha\sigma)\phi e_P^* - \\ R_E^* - (1 - \beta_E^*)(\mu + \alpha\sigma)\phi e_E^*. \end{aligned} \tag{28}$$

The solution of Eq. (28) yields the following results (see the Appendix for details).

Proposition 9. *Let Assumption 3 hold. Then there exist a loading factor* $\lambda^* \in (\widehat{\lambda}, \overline{\lambda})$ *such that the Board of Directors adopts an ECSR statute for* $\lambda > \lambda^*$.

The result in Proposition 9 is central to this paper, as it shows that engaging in ECSR activities becomes a strategic choice to counter an adverse insurance market, represented by the presence of a high loading factor. The high loading factor is reflected in a higher insurance premium in equilibrium. If the firm becomes ECSR, it reduces more emissions compared to the alternative PS statute, by thus counterbalancing the negative effect of the high loading factor into the premium level.

5 Concluding remarks

We have studied the interplay between demand of insurance against environmental accidents and the adoption of ECSR practices. Being a commitment to voluntary reduce the polluting emissions, ECSR activities interact with environmentally concerned consumers and the tax on emissions. In addition, given that the environmental accidents are influenced by the level of emissions, it may also act as a self-insurance tool.

Our results have shown that the adoption of ECSR may be driven by a tight insurance market, with a high loading factor. The extra emission reduction implicit in the ECSR activities may offset the effect of the high loading factor in the insurance premium.

One possible avenue for future research is to extend the framework to include the insurer's decision-making process. For example, we could consider how insurers price policies under different regulatory constraints or capital requirements. Indeed, insurance premiums are a key driver of a firm's decision to engage in ECSR activities. Examining how insurers respond to different risk profiles or market regulations would provide a richer, two-sided view of the link between ECSR and insurance.

In addition to an emissions tax, alternative or complementary policy instruments could be explored: cap-and-trade systems, subsidies for green technology, stricter liability rules or mandatory insurance. Different policy instruments alter both the cost of pollution and the firm's perception of environmental risk. Comparing outcomes under different policy regimes - especially for risk-averse firms - would give policymakers clearer guidance on which instruments best stimulate investment in ECSR while ensuring adequate insurance coverage.

The analysis of the insurer's decision-making process and the comparison of different policy instruments are interesting extensions left for future research.

Appendix

Complete derivation of market stage: PS firm

The Lagrangian function associated with Eq. (13) is

$$\mathcal{L}_P = \pi_P + \nu_1 q_P + \nu_2 z_P + (q_P - z_P)\nu_3,$$

where $\nu_1,\ \nu_2,\ \nu_3 \geqslant 0$ are the Kuhn–Tucker multipliers. The optimality conditions of the constrained optimization are

$$\begin{cases} \gamma - 2q_P - \tau - (1-\beta_P)(\mu + \alpha\sigma)\phi + \nu_1 + \nu_3 = 0 \\ -z_P + \tau + (1-\beta_P)(\mu + \alpha\sigma)\phi + \nu_2 - \nu_3 = 0 \\ \nu_1 q_E = 0,\ \nu_1 \geqslant 0 \\ \nu_2 z_E = 0,\ \nu_2 \geqslant 0 \\ (q_P - z_P)\nu_3 = 0,\ \nu_3 \geqslant 0 \\ q_P \geqslant 0,\ z_P \geqslant 0,\ q_P - z_P \geqslant 0 \end{cases} \tag{29}$$

Solving Eq. (29) with respect to q_P and z_P and defining

$$\widehat{\gamma}_P \equiv 3[\tau + (1-\beta_P)(\mu + \alpha\sigma)\phi],$$

we obtain the following optimal values:

$$q_P^* = \begin{cases} \dfrac{1}{3}\gamma & \text{if } \gamma \leqslant \widehat{\gamma}_P, \\ \dfrac{1}{2}[\gamma - \tau - (1-\beta_P)(\mu + \alpha\sigma)\phi] & \text{if } \gamma > \widehat{\gamma}_P, \end{cases}$$
$$z_P^* = \begin{cases} \dfrac{1}{3}\gamma & \text{if } \gamma \leqslant \widehat{\gamma}_P, \\ \tau + (1-\beta_P)(\mu + \alpha\sigma)\phi & \text{if } \gamma > \widehat{\gamma}_P. \end{cases}$$

Proof of Corollary 2

Differentiating β_P^* with respect to γ, τ and ϕ, respectively, yields

$$\begin{aligned}
\frac{\partial \beta_P^*}{\partial \gamma} &= \frac{\alpha\sigma - \lambda\mu}{3(\mu+\alpha\sigma)(\mu-\alpha\sigma+2\lambda\mu)\phi} > 0,\\
\frac{\partial \beta_P^*}{\partial \tau} &= -\frac{\alpha\sigma - \lambda\mu}{(\mu+\alpha\sigma)(\mu-\alpha\sigma+2\lambda\mu)\phi} < 0,\\
\frac{\partial \beta_P^*}{\partial \phi} &= -\frac{(\alpha\sigma - \lambda\mu)(\gamma-3\tau)}{3(\mu+\alpha\sigma)(\mu-\alpha\sigma+2\lambda\mu)\phi^2} < 0,\\
\frac{\partial \beta_P^*}{\partial \lambda} &= -\frac{[\gamma-3\tau-3(\mu+\alpha\sigma)\phi]\mu}{3(\mu-\alpha\sigma+2\lambda\mu)^2\phi} < 0.
\end{aligned}$$

□

Proof of Proposition 3

Differentiation of equilibrium profits with respect to the tax on emissions, yields

$$\begin{aligned}
\frac{\partial \pi_P^*}{\partial \tau} &= 2\frac{\partial q_P^*}{\partial \tau}q_P^* - \frac{\partial z_P^*}{\partial \tau}z_P^* + z_P^* + \frac{\partial z_P^*}{\partial \tau}\tau + (\mu+\alpha\sigma)\left[(1-\beta_P^*)\frac{\partial z_P^*}{\partial \tau} - \frac{\partial \beta_P^*}{\partial \tau}z_P^*\right]\phi\\
&= \left[(\mu+\alpha\sigma)\frac{\partial \beta_P^*}{\partial \tau}\phi - 1\right]q_P^* - (\mu+\alpha\sigma)\frac{\partial \beta_P^*}{\partial \tau}\phi z_P^* + z_P^*\\
&= (q_P^* - z_P^*)(\mu+\alpha\sigma)\frac{\partial \beta_P^*}{\partial \tau}\phi - (q_P^* - z_P^*). \qquad (30)
\end{aligned}$$

A close inspection of Eq. (30) shows that the derivative is negative, given that

$$\begin{aligned}
\tau + (1-\beta_P^*)(\mu+\alpha\sigma)\phi - z_P^* &= 0,\\
\frac{\partial \beta_P^*}{\partial \tau} &= -\frac{\alpha\sigma - \lambda\mu}{(\mu+\alpha\sigma)(\mu-\alpha\sigma+2\lambda\mu)\phi} < 0,
\end{aligned}$$

and $q_P^* - z_P^* > 0$.

Differentiation of equilibrium profits with respect to the impact of emissions over the financial loss yields

$$
\begin{aligned}
\frac{\partial \pi_P^*}{\partial \phi} &= 2\frac{\partial q_P^*}{\partial \phi} q_P^* - \frac{\partial z_P^*}{\partial \phi} z_P^* + \frac{\partial z_P^*}{\partial \phi}\tau + (\mu+\alpha\sigma)\left[(1-\beta_P^*) z_P^* + (1-\beta_P^*)\frac{\partial z_P^*}{\partial \phi}\phi - \frac{\partial \beta_P^*}{\partial \phi}\phi z_P^*\right], \\
&= (\mu+\alpha\sigma)\left[\frac{\partial \beta_P^*}{\partial \phi}\phi - (1-\beta_P^*)\right] q_P^* + (1-\beta_P^*)(\mu+\alpha\sigma) z_P^* - (\mu+\alpha\sigma)\frac{\partial \beta_P^*}{\partial \phi}\phi z_P^*, \\
&= (\mu+\alpha\sigma)(q_P^* - z_P^*)\frac{\partial \beta_P^*}{\partial \phi}\phi - (1-\beta_P^*)(\mu+\alpha\sigma)(q_P^* - z_P^*). \qquad (31)
\end{aligned}
$$

Eq. (31) is negative, since

$$
\begin{aligned}
\tau + (1-\beta_P^*)(\mu+\alpha\sigma)\phi - z_P^* &= 0, \\
\frac{\partial \beta_P^*}{\partial \phi} &= -\frac{(\alpha\sigma - \lambda\mu)(\gamma - 3\tau)}{3(\mu+\alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi^2} < 0,
\end{aligned}
$$

and $q_P^* - z_P^* > 0$.

Finally, differentiation of equilibrium profits with respect to the loading factor yields

$$
\begin{aligned}
\frac{\partial \pi_P^*}{\partial \lambda} &= 2\frac{\partial q_P^*}{\partial \lambda} q_P^* - \frac{\partial z_P^*}{\partial \lambda} z_P^* + \frac{\partial z_P^*}{\partial \lambda}\tau + (\mu+\alpha\sigma)\left[(1-\beta_P^*)\frac{\partial z_P^*}{\partial \lambda}\phi - \frac{\partial \beta_P^*}{\partial \lambda} z_P^*\right]\phi, \\
&= (\mu+\alpha\sigma)\frac{\partial \beta_P^*}{\partial \lambda}\phi q_P^* - (\mu+\alpha\sigma)\frac{\partial \beta_P^*}{\partial \lambda}\phi z_P^*, \\
&= (\mu+\alpha\sigma)(q_P^* - z_P^*)\frac{\partial \beta_P^*}{\partial \lambda}\phi. \qquad (32)
\end{aligned}
$$

Again, Eq. (32) is negative, given that

$$
\begin{aligned}
\tau + (1-\beta_P^*)(\mu+\alpha\sigma)\phi - z_P^* &= 0, \\
\frac{\partial \beta_P^*}{\partial \lambda} &= -\frac{[\gamma - 3\tau - 3(\mu+\alpha\sigma)\phi]\mu}{3(\mu - \alpha\sigma + 2\lambda\mu)^2\phi} < 0,
\end{aligned}
$$

and $q_P^* - z_P^* > 0$.

$\square$

Complete derivation of market stage: ECSR firm

The Lagrangian function associated with the problem Eq. (20) is

$$\mathcal{L}_E = O_E + \nu_4 q_E + \nu_5 z_E + (q_E - z_E)\nu_6 \tag{33}$$

where $\nu_4,\ \nu_5,\ \nu_6 \geqslant 0$ are the Kuhn–Tucker multipliers. From Eq. (33), we obtain the optimality conditions of the constrained optimization:

$$\begin{cases} \gamma + \theta\eta - 2q_E - \tau - \eta - (1-\beta_E)(\mu + \alpha\sigma)\phi + \nu_4 + \nu_6 = 0 \\ -z_E + \tau + \eta + (1-\beta_E)(\mu + \alpha\sigma)\phi + \nu_5 - \nu_6 = 0 \\ \nu_4 q_E = 0,\ \nu_4 \geqslant 0 \\ \nu_5 z_E = 0,\ \nu_5 \geqslant 0 \\ (q_E - z_E)\nu_6 = 0,\ \nu_6 \geqslant 0 \\ q_E \geqslant 0,\ z_E \geqslant 0,\ q_E - z_E \geqslant 0 \end{cases} \tag{34}$$

Solving Eq. (34) with respect to q_E and z_E and defining

$$\widehat{\gamma}_E = (3-\theta)\eta + 3[\tau + (\mu + \alpha\sigma)\phi],$$

we obtain the following optimal values:

$$q_E^* = \begin{cases} \frac{1}{3}\gamma & \text{if } \gamma \leqslant \widehat{\gamma}_E, \\ \frac{1}{2}[\gamma - \tau - (1-\beta)(\mu + \alpha\sigma)\phi] & \text{if } \gamma > \widehat{\gamma}_E, \end{cases}$$
$$z_E^* = \begin{cases} \frac{1}{3}\gamma & \text{if } \gamma \leqslant \widehat{\gamma}_E, \\ \tau + (1-\beta)(\mu + \alpha\sigma)\phi & \text{if } \gamma > \widehat{\gamma}_E. \end{cases}$$

Proof of Corollary 3

Differentiation of β_E^* with respect to γ, τ and ϕ, respectively, yields

$$\begin{aligned}
\frac{\partial \beta_E^*}{\partial \gamma} &= \frac{\alpha\sigma - \lambda\mu}{3(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi} > 0, \\
\frac{\partial \beta_E^*}{\partial \tau} &= -\frac{\alpha\sigma - \lambda\mu}{(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi} < 0, \\
\frac{\partial \beta_E^*}{\partial \phi} &= -\frac{(\alpha\sigma - \lambda\mu)[\gamma - 3\tau - (1-\theta)\eta]}{3(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi^2} < 0, \\
\frac{\partial \beta_E^*}{\partial \lambda} &= -\frac{[\gamma - (3-\theta)\eta - 3\tau - 3(\mu + \alpha\sigma)\phi]\mu}{3(\mu - \alpha\sigma + 2\lambda\mu)^2\phi} < 0.
\end{aligned}$$

□

Proof of Corollary 4

Differentiating of β_E^* with respect to θ and η, respectively, yields

$$\begin{aligned}
\frac{\partial \beta_E^*}{\partial \theta} &= \frac{(\alpha\sigma - \lambda\mu)\eta}{3(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi} > 0, \\
\frac{\partial \beta_E^*}{\partial \eta} &= -\frac{(\alpha\sigma - \lambda\mu)(3-\theta)}{3(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi} < 0.
\end{aligned}$$

□

Proof of Proposition 6

Differentiation of the ECSR equilibrium profits with respect to the tax on emissions, yields

$$\begin{aligned}
\frac{\partial \pi_E^*}{\partial \tau} &= (2q_E^* + \eta)\frac{\partial q_E^*}{\partial \tau} - \frac{\partial z_E^*}{\partial \tau} z_E^* + z_E^* + \frac{\partial z_E^*}{\partial \tau}\tau + (\mu + \alpha\sigma)\left[(1 - \beta_E^*)\frac{\partial z_E^*}{\partial \tau} - \frac{\partial \beta_E^*}{\partial \tau}\right]\phi, \\
&= \frac{1}{2}(2q_E^* + \eta)\left[(\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \tau}\phi - 1\right] - (\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \tau}\phi z_E^* - \frac{\partial z_E^*}{\partial \tau}\eta + z_E^*, \\
&= \left(q_E^* - z_E^* + \frac{\eta}{2}\right)(\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \tau}\phi - \left(q_E^* - z_E^* + \frac{\eta}{2}\right) - \frac{\partial z_E^*}{\partial \tau}\eta. \qquad (35)
\end{aligned}$$

A close inspection of Eq. (35) shows that the derivative is negative, given that

$$\tau + (1-\beta_E^*)(\mu+\alpha\sigma)\phi - z_E^* = -\eta,$$
$$\frac{\partial \beta_E^*}{\partial \tau} = -\frac{\alpha\sigma - \lambda\mu}{(\mu+\alpha\sigma)(\mu-\alpha\sigma+2\lambda\mu)\phi} < 0,$$
$$\frac{\partial z_E^*}{\partial \tau} = \frac{(1+\lambda)\mu}{\mu-\alpha\sigma+2\lambda\mu} > 0,$$

and $q_E^* - z_E^* > 0$.

Differentiation of the ECSR equilibrium profits with respect to the impact of emissions over the financial loss yields

$$\begin{aligned}
\frac{\partial \pi_E^*}{\partial \phi} &= (2q_E^*+\eta)\frac{\partial q_E^*}{\partial \phi} - \frac{\partial z_E^*}{\partial \phi} z_E^* + \frac{\partial z_E^*}{\partial \phi}\tau + (\mu+\alpha\sigma)\left[(1-\beta_E^*)z_E^* - \frac{\partial \beta_E^*}{\partial \phi}\phi z_E^* + (1-\beta_E^*)\frac{\partial z_E^*}{\partial \phi}\phi\right], \\
&= \frac{1}{2}(2q_E^*+\eta)(\mu+\alpha\sigma)\left[\frac{\partial \beta_E^*}{\partial \phi}\phi - (1-\beta_E^*)\right] + (1-\beta_E^*)(\mu+\alpha\sigma)z_E^* \\
&- (\mu+\alpha\sigma)\frac{\partial \beta_E^*}{\partial \phi}\phi z_E^* - \frac{\partial z_E^*}{\partial \phi}\eta, \\
&= (\mu+\alpha\sigma)\left(q_E^* - z_E^* + \frac{\eta}{2}\right)\left[\frac{\partial \beta_E^*}{\partial \phi}\phi - (1-\beta_E^*)\right] - \frac{\partial z_E^*}{\partial \phi}\eta. \qquad (36)
\end{aligned}$$

Eq. (36) is negative, since

$$\tau + (1-\beta_E^*)(\mu+\alpha\sigma)\phi - z_E^* = -\eta,$$
$$\frac{\partial \beta_E^*}{\partial \phi} = -\frac{(\alpha\sigma - \lambda\mu)(\gamma-(1-\theta)\eta-3\tau)}{3(\mu+\alpha\sigma)(\mu-\alpha\sigma+2\lambda\mu)\phi^2} < 0,$$
$$\frac{\partial z_E^*}{\partial \phi} = \frac{(1+\lambda)(\mu+\alpha\sigma)\mu}{\mu-\alpha\sigma+2\lambda\mu} > 0,$$

and $q_E^* - z_E^* > 0$.

Finally, differentiation of ECSR equilibrium profits with respect to the loading factor yields

$$\begin{aligned}\frac{\partial \pi_E^*}{\partial \lambda} &= (2q_E^* + \eta)\frac{\partial q_E^*}{\partial \lambda} - \frac{\partial z_E^*}{\partial \lambda}z_E^* + \frac{\partial z_E^*}{\partial \lambda}\tau + (\mu + \alpha\sigma)\left[(1-\beta_E^*)\frac{\partial z_E^*}{\partial \lambda} - \frac{\partial \beta_E^*}{\partial \lambda}z_E^*\right]\phi,\\ &= \frac{1}{2}(2q_E^* + \eta)(\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \lambda}\phi - (\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \lambda}\phi z_E^* - \frac{\partial z_E^*}{\partial \lambda}\eta,\\ &= \frac{1}{2}(\mu + \alpha\sigma)(2q_E^* + \eta - z_E^*)\frac{\partial \beta_E^*}{\partial \lambda}\phi - \frac{\partial z_E^*}{\partial \lambda}\eta. \end{aligned} \tag{37}$$

Again, Eq. (37) is negative, given that

$$\tau + (1-\beta_E^*)(\mu + \alpha\sigma)\phi - z_E^* = -\eta,$$

$$\frac{\partial \beta_E^*}{\partial \lambda} = -\frac{[\gamma - (3-\theta)\eta - 3\tau - 3(\mu + \alpha\sigma)\phi]\mu}{3(\mu - \alpha\sigma + 2\lambda\mu)^2\phi} < 0,$$

$$\frac{\partial z_E^*}{\partial \phi} = \frac{(\mu + \alpha\sigma)[\gamma - (3-\theta)\eta - 3\tau - 3(\mu + \alpha\sigma)\phi]\mu}{3(\mu - \alpha + 2\lambda\mu)^2} > 0,$$

and $q_E^* - z_E^* > 0$. □

Proof of Proposition 7

Differentiation of the ECSR equilibrium profits with respect to consumers' environmental concern yields

$$\begin{aligned}\frac{\partial \pi_E^*}{\partial \theta} &= (2q_E^* + \eta)\frac{\partial q_E^*}{\partial \theta} + [\tau + (1-\beta_E^*)(\mu + \alpha\sigma)\phi - z_E^*]\frac{\partial z_E^*}{\partial \theta} - (\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \theta}\phi z_E^*,\\ &= \frac{1}{2}(2q_E^* + \eta)\left[\eta + (\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \theta}\phi\right] - (\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \theta}\phi z_E^* - \frac{\partial z_E^*}{\partial \theta}\eta,\\ &= \left(q_E^* + \frac{\eta}{2}\right)\eta + (\mu + \alpha\sigma)\left(q_E^* - z_E^* + \frac{\eta}{2}\right)\frac{\partial \beta_E^*}{\partial \theta}\phi - \frac{\partial z_E^*}{\partial \theta}\eta. \end{aligned} \tag{38}$$

Eq. (38) is positive given that

$$\tau + (1-\beta_E^*)(\mu + \alpha\sigma)\phi - z_E^* = -\eta,$$

$$\frac{\partial z_E^*}{\partial \theta} = -\frac{(\alpha\sigma - \lambda\mu)\eta}{3(\mu - \alpha\sigma + 2\lambda\mu)} < 0,$$

$$\frac{\partial q_E^*}{\partial \theta} = \eta + (\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \theta}\phi > 0,$$

$$\frac{\partial \beta_E^*}{\partial \theta} = \frac{(\alpha\sigma - \lambda\mu)\eta}{3(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi} > 0,$$

and $q_E^* - z_E^* > 0$.

Finally, differentiation of the ECSR equilibrium profits with respect to emission internalisation yields

$$\begin{aligned}
\frac{\partial \pi_E^*}{\partial \eta} &= (2q_E^* + \eta)\frac{\partial q_E^*}{\partial \eta} + q_E^* + [\tau + (1 - \beta_E^*)(\mu + \alpha\sigma)\phi - z_E^*]\frac{\partial z_E^*}{\partial \eta} - (\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \eta}\phi z_E^*, \\
&= q_E^* + \left(q_E^* + \frac{\eta}{2}\right)\left[(\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \eta}\phi - (1 - \theta)\right] - (\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \eta}\phi z_E^* - \frac{\partial z_E^*}{\partial \eta}\eta, \\
&= q_E^* + (\mu + \alpha\sigma)\left(q_E^* - z_E^* + \frac{\eta}{2}\right)\frac{\partial \beta_E^*}{\partial \eta}\phi - (1 - \theta)\left(q_E^* + \frac{\eta}{2}\right) - \frac{\partial z_E^*}{\partial \eta}\eta, \\
&= (\mu + \alpha\sigma)\left(q_E^* - z_E^* + \frac{\eta}{2}\right)\frac{\partial \beta_E^*}{\partial \eta}\phi - \frac{\eta}{2} - \left(q_E^* + \frac{\eta}{2}\right)\theta - \frac{\partial z_E^*}{\partial \eta}\eta. \qquad (39)
\end{aligned}$$

Eq. (39) is negative since

$$\tau + (1 - \beta_E^*)(\mu + \alpha\sigma)\phi - z_E^* = -\eta,$$

$$\frac{\partial z_E^*}{\partial \eta} = 1 + \frac{(3 - \theta)(\alpha\sigma)}{3(\mu - \alpha\sigma + 2\lambda\mu)} > 0,$$

$$\frac{\partial q_E^*}{\partial \eta} = \frac{1}{2}\left[(\mu + \alpha\sigma)\frac{\partial \beta_E^*}{\partial \eta}\phi - (1 - \theta)\right] < 0,$$

$$\frac{\partial \beta_E^*}{\partial \eta} = -\frac{(\alpha\sigma - \lambda\mu)(3 - \theta)}{3(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi} < 0,$$

and $q_E^* - z_E^* > 0$. □

Proof of Proposition 8

Suppose Assumption 3 holds. Comparing equilibrium quantities between a firm of PS and ECSR type, one gets

$$q_P^* - q_E^* = \frac{[3(1 - \theta)\mu + (3 - 5\theta)\lambda\mu + 2\theta\alpha\sigma]\eta}{6(\mu - \alpha\sigma + 2\lambda\mu)}. \qquad (40)$$

Eq. (40) is positive as long as

$$3(1-\theta)\mu + (3-5\theta)\lambda\mu + 2\theta\alpha\sigma > 0,$$

which occurs for

$$\lambda < \frac{3(1-\theta)\mu + 2\theta\alpha\sigma}{(5-3\theta)\mu}.$$

Since

$$\overline{\lambda} - \frac{3(1-\theta)\mu + 2\theta\alpha\sigma}{(5-3\theta)\mu} = \frac{(5\alpha\sigma - 3\mu)\,(1-\theta)}{(5-3\theta)\,\mu} > 0,$$

then $q_P^* > q_E^*$ given Assumption 3.

Comparing the equilibrium investment in emission reduction between a PS and an ECSR firm, we obtain

$$z_P^* - z_E^* = -\frac{[3(1+\lambda)\mu - (\alpha\sigma - \lambda\mu)\theta]\eta}{3(\mu - \alpha\sigma + 2\lambda\mu)}. \tag{41}$$

Eq. (41) is negative as long as

$$3(1+\lambda)\mu - (\alpha\sigma - \lambda\mu)\theta,$$

which is positive for

$$\lambda > \frac{\theta\alpha\sigma - 3\mu}{(3+\theta)\mu}$$

Given that

$$\widehat{\lambda} - \frac{\theta\alpha\sigma - 3\mu}{(3+\theta)\mu} = \frac{(\mu + \alpha\sigma)\,(3-\theta)}{2\,(\theta+3)\,\mu} > 0,$$

then $z_P^* < z_E^*$ given Assumption 3.

Comparing the equilibrium level of emissions between the PS and ECSR firm, one gets

$$e_P^* - e_E^* = \frac{(1+\lambda)(3-\theta)\eta\mu}{2(\mu - \alpha\sigma + 2\lambda\mu)} > 0. \tag{42}$$

Finally, comparing the equilibrium fraction of insurance coverage,

$$\beta_P^* - \beta_E^* = \frac{(3-\theta)(\alpha\sigma - \lambda\mu)\eta}{3(\mu + \alpha\sigma)(\mu - \alpha\sigma + 2\lambda\mu)\phi} < 0. \tag{43}$$

Proof of Proposition 9

Suppose Assumption 3 holds. From Proposition 3 and Proposition 6, we can derive that $\frac{\partial \pi_P^*}{\partial \lambda} - \frac{\partial \pi_E^*}{\partial \lambda}$ corresponds to

$$\begin{aligned}\frac{\partial \pi_P^*}{\partial \lambda} - \frac{\partial \pi_E^*}{\partial \lambda} =& \frac{1}{6(\mu - \alpha\sigma + 2\lambda\mu)^3} \times \\ & \{[(2\gamma + \theta\eta - 6\tau - 6(\mu + \alpha\sigma)\phi)\theta - 9\eta]\lambda - 3(\mu + \alpha\sigma)(\gamma + \theta\eta - 3\tau + 2\phi\mu\theta) + \\ & (2\gamma + \theta^2\eta - 6\theta\tau)\mu - 9(\mu^2 - \alpha^2\sigma^2)\phi + 9(2\mu + \eta)\alpha\sigma\},\end{aligned}$$

which is negative for $\lambda > \lambda_1$, where

$$\lambda_1 \equiv \frac{3(\mu + \alpha\sigma)(\gamma + \theta\eta - 3\tau + 2\phi\mu\theta) - (2\gamma + \theta^2\eta - 6\theta\tau)\mu + 9(\mu^2 - \alpha^2\sigma^2)\phi - 9(2\mu + \eta)\alpha\sigma}{[2\gamma + \theta\eta - 6\tau - 6(\mu + \alpha\sigma)\phi]\theta - 9\eta}.$$

Given that

$$\lambda_1 - \widehat{\lambda} = -\frac{(3 - \theta)(\mu + \alpha\sigma)(2\gamma - (3 - \theta)\eta - 6\tau - 6(\mu + \alpha\sigma)\phi)}{2[2\gamma + \theta\eta - 6\tau - 6(\mu + \alpha\sigma)\phi]\mu} < 0,$$

then $\frac{\partial \pi_P^*}{\partial \lambda} < \frac{\partial \pi_E^*}{\partial \lambda}$ given Assumption 3. Moreover, both π_E^* and π_P^* are convex function in λ:

$$\begin{aligned}\frac{\partial^2 \pi_E^*}{\partial \lambda^2} &= (\mu + \alpha\sigma)\left[\left(\frac{\partial q_E^*}{\partial \lambda} - \frac{\partial z_E^*}{\partial \lambda}\right)\frac{\partial \beta_E^*}{\partial \lambda} + \left(q_E^* - z_E^* + \frac{\eta}{2}\right)\frac{\partial^2 \beta_E^*}{\partial \lambda^2}\right]\phi - \frac{\partial^2 z_E^*}{\partial \lambda^2}\eta > 0, \\ \frac{\partial^2 \pi_P^*}{\partial \lambda^2} &= (\mu + \alpha\sigma)\left[\left(\frac{\partial q_P^*}{\partial \lambda} - \frac{\partial z_P^*}{\partial \lambda}\right)\frac{\partial \beta_P^*}{\partial \lambda} + (q_P^* - z_P^*)\frac{\partial^2 \beta_P^*}{\partial \lambda^2}\right]\phi > 0.\end{aligned}$$

In addition $\frac{\partial \beta_P^*}{\partial \lambda} < 0$, $\frac{\partial \beta_E^*}{\partial \lambda} < 0$, and

$$\frac{\partial q_E^*}{\partial \lambda} - \frac{\partial z_E^*}{\partial \lambda} = -\frac{(\mu + \alpha\sigma)[\gamma - (3 - \theta)\eta - 3\tau - 3(\mu + \alpha\sigma)\phi]\mu}{2(\mu - \alpha\sigma + 2\lambda\mu)^2} < 0,$$

$$\frac{\partial^2 \beta_E^*}{\partial \lambda^2} = \frac{4[\gamma - (3 - \theta)\eta - 3\tau - 3(\mu + \alpha\sigma)\phi]\mu^2}{3(\mu - \alpha\sigma + 2\lambda\mu)^3\phi} > 0,$$

$$\frac{\partial^2 z_E^*}{\partial \lambda^2} = -\frac{4(\mu + \alpha\sigma)[\gamma - (3 - \theta)\eta - 3\tau - 3(\mu + \alpha\sigma)\phi]\mu^2}{3(\mu - \alpha\sigma + 2\lambda\mu)^3} < 0,$$

$$\frac{\partial q_P^*}{\partial \lambda} - \frac{\partial z_P^*}{\partial \lambda} = -\frac{(\mu + \alpha\sigma)[\gamma - 3\tau - 3(\mu + \alpha\sigma)\phi]\mu}{2(\mu - \alpha\sigma + 2\lambda\mu)^2},$$

$$\frac{\partial^2 \beta_P^*}{\partial \lambda^2} = \frac{\mu}{(\mu - \alpha\sigma + 2\lambda\mu)^2\phi} > 0.$$

Then there exists an intersection point λ^*, such that $\pi_P^* = \pi_E^*$. If $\lambda^* < \underline{\lambda}$, with $\underline{\lambda} = \max\{\underline{\lambda}_E, \underline{\lambda}_P\}$, then $\pi_P^* - \pi_E^* < 0$ for every λ. If $\lambda^* \in (\underline{\lambda}, \overline{\lambda})$ then $\pi_P^* - \pi_E^* < 0$ for every $\lambda \in (\underline{\lambda}, \lambda^*)$ and $\pi_P^* - \pi_E^* > 0$, for every $\lambda \in (\lambda^*, \overline{\lambda})$. Finally, if $\lambda^* > \overline{\lambda}$, then $\pi_P^* - \pi_E^* > 0$ for every λ. $\square$

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