Debt Value and Capital Structure with Firm's Net Cash Payouts

Flavia Barsotti
Dept. Stat. and Applied Math.
University of Pisa, Italy
f.barsotti@ec.unipi.it

Maria Elvira Mancino
Dept. Math. for Decisions
University of Firenze, Italy
mariaelvira.mancino@dmd.unifi.it

Monique Pontier
Inst. Math. de Toulouse (IMT)
University of Toulouse, France
pontier@math.univ-toulouse.fr

September 3, 2010

Abstract

In this paper a structural model of corporate debt is analyzed following an approach of optimal stopping problem. We extend Leland model [5] introducing a dividend δ paid to equity holders and studying its effect on corporate debt and optimal capital structure. Varying the parameter δ affects not only the level of endogenous bankruptcy, which is decreased, but modifies the magnitude of a change on the endogenous failure level as a consequence of an increase in risk free rate, corporate tax rate, riskiness of the firm and coupon payments. Concerning the optimal capital structure, the introduction of dividends allows to obtain results more in line with historical norms: lower optimal leverage ratios and higher yield spreads, compared to Leland's [5] results.

1 Introduction

Many firm value models have been proposed since Merton's work [8] which provides an analytical framework in which the capital structure of a firm is analyzed in terms of derivatives contracts. We focus on the corporate model proposed by Leland [5] assuming that the firm's assets value evolves as a geometric Brownian motion. According to Modigliani - Miller theorem [9] the activities of the firm are independent from the financial structure. The firm realizes its capital from both debt and equity. Debt is perpetual, it pays a constant coupon C per instant of time and this determines tax benefits proportional to coupon payments. Bankruptcy is determined endogenously by the inability of the firm to raise sufficient equity capital to cover its debt obligations. On the failure time T, agents which hold debt claims will get the residual value of the firm (because of bankruptcy costs), and those who hold equity will get nothing (the strict priority rule holds). This paper examines the case where the firm has net cash outflows resulting from payments to bondholders or stockholders, for instance if dividends are paid to equity holders. The interest in this problem is posed in [5] section VI-B, nevertheless the resulting optimal capital structure is not analyzed in detail. The aim of this note is twofold: from one hand we complete the study of corporate debt and optimal leverage in the presence of dividends in all analytical aspects, from the other hand we study numerically the effects of this variation on the capital structure. We will follow Leland [5] by considering only cash outflows which are proportional to firm's assets value but our analysis differs from Leland's one since we solve the optimal control problem as an optimal stopping problem (see also [2] for a similar approach). We find that the increase of the dividend parameter δ affects not only the level of endogenous bankruptcy, which is decreased, but modifies the magnitude of a change on the endogenous failure level as a consequence of an increase in risk free rate, corporate tax rate, riskiness of the firm and coupon payments. Further the introduction of dividends allows to obtain lower optimal leverage ratios and higher yield spreads, compared to Leland's [5] results.

The paper is organized as follows: Section 2 introduces the model and determines the optimal failure time as an optimal stopping time, getting the endogenous failure level. Then, the influence of coupon, dividend and corporate tax rate on all financial variables is studied. Section 4 describes optimal capital structure as a consequence of optimal coupon choice.

2 Capital Structure Model with Dividends

In this section we introduce the model, which is very close to Leland's [5], but we modify the drift with a parameter δ, which might represent a constant proportional cash flow generated by the assets and distributed to security holders. A firm realizes its capital from both debt and equity. Debt is perpetual and pays a constant coupon C per instant of time. On the failure time T, agents which hold debt claims will get the residual value of the firm, and those who hold equity will get nothing. We assume that the firm activities value is described by the process $V_t = Ve^{X_t}$, where X_t evolves, under the risk neutral probability measure, as

$$dX_t = \left(r - \delta - \frac{1}{2}\sigma^2 \right) dt + \sigma dW_t, \;\; X_0 = 0, \tag{1}$$

where W is a standard Brownian motion, r the constant risk-free rate, r, δ and $\sigma > 0$. When bankruptcy occurs at stopping time T, a fraction α $(0 \le \alpha < 1)$ of firm value is lost (for instance payed because of bankruptcy procedures), debt holders receive the rest and stockholders nothing, meaning that the strict priority rule holds. We suppose that the failure time T is a stopping time. Thus, applying contingent claim analysis in a Black-Scholes setting, for a given stopping (failure) time T, *debt value* is

$$D(V, C, T) = \mathbb{E}_V \left[\int_0^T e^{-rs} C ds + (1 - \alpha) e^{-rT} V_T \right], \tag{2}$$

where the expectation is taken with respect to the risk neutral probability and we denote

$$\mathbb{E}_V[\cdot] := \mathbb{E}[\cdot | V_0 = V].$$

We assume that from paying coupons the firm obtains tax deductions, namely τ, $0 \le \tau < 1$, proportionally to coupon payments, so we get *equity value*

$$E(V, C, T) = V - \mathbb{E}_V \left[(1 - \tau) \left(\int_0^T e^{-rs} C ds \right) + e^{-rT} V_T \right]. \tag{3}$$

The *total value* of the (levered) firm can always be expressed as sum of equity and debt value: this leads to write the total value of the levered firm as the firm's asset value (unlevered) plus tax deductions on debt payments C less the value of bankruptcy costs:

$$v(V, C, T) = V + \mathbb{E}_V \left[\tau \int_0^T e^{-rs} C ds - \alpha e^{-rT} V_T \right]. \tag{4}$$

2.1 Endogenous Failure Level

On the set of stopping times we maximize the equity value $T \mapsto E(V, C, T)$, for an arbitrary level of the coupon rate C. By optimal stopping theory (see [3]) and following [1, 2], the failure time, "optimal stopping time", is known to be a constant level hitting time. Hence default happens at passage time T when the value $V_\cdot$ falls to some constant level V_B. The value of V_B is endogenously derived and will be determined with an optimal rule later. Further we note that, given (1), it holds that $T = \inf\{t \geq 0 : \ V_t \leq V_B\} = \inf\{t \geq 0 : \ X_t \leq \log \frac{V_B}{V}\}$. Moreover it holds $V_T = V_B$, as the process V_t is continuous.

Thus, the optimal stopping problem of equity holders is turned to maximize the equity function defined in (3) as a function of V_B:

$$E : V_B \mapsto V - \frac{(1-\tau)C}{r}(1 - \mathbb{E}_V[e^{-rT}]) - V_B \mathbb{E}_V[e^{-rT}]. \tag{5}$$

In order to compute the equity value (5) it remains to determine $\mathbb{E}_V \left[e^{-rT}\right]$. To this hand we use the following formula for the Laplace transform of a constant level hitting time by a Brownian motion with drift ([4] p. 196-197):

Proposition 2.1 *Let $X_t = \mu t + \sigma W_t$ and $T_b = \inf\{s : X_s = b\}$, then, for all $\gamma > 0$, it holds*

$$\mathbb{E}[e^{-\gamma T_b}] = \exp\left[\frac{\mu b}{\sigma^2} - \frac{|b|}{\sigma} \sqrt{\frac{\mu^2}{\sigma^2} + 2\gamma} \right].$$

Since $V_t = V \exp[(r - \delta - \frac{1}{2}\sigma^2)t + \sigma W_t]$ by (1), we get $\mathbb{E}_V[e^{-rT}] = \left(\frac{V_B}{V}\right)^{\lambda(\delta)}$ where

$$\lambda(\delta) = \frac{r - \delta - \frac{1}{2}\sigma^2 + \sqrt{(r - \delta - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}. \tag{6}$$

Remark 2.2 *As a function of δ, the coefficient $\lambda(\delta)$ in (6) is decreasing and convex. In order to simplify the notation, we will denote $\lambda(\delta)$ as λ in the sequel.*

Finally the equity function to be optimized w.r.t. V_B is

$$E : V_B \mapsto V - \frac{(1-\tau)C}{r} + \left(\frac{(1-\tau)C}{r} - V_B\right)\left(\frac{V_B}{V}\right)^{\lambda}, \tag{7}$$

and the following properties must be satisfied:

$$E(V, C, T) \geq E(V, C, \infty) \quad \text{and} \quad E(V, C, T) \geq 0 \quad \text{for all } V \geq V_B. \tag{8}$$

Considering the equity function in (7), the first property in (8) is equivalent to

$$E(V,C,T) - E(V,C,\infty) = \left(\frac{(1-\tau)C}{r} - V_B\right)\left(\frac{V_B}{V}\right)^{\lambda} \geq 0.$$

In fact this term is the option embodied in equity. Since this is an option to be exercised by the firm, this must have positive value, so it must be $\frac{(1-\tau)C}{r} - V_B \geq 0$. Finally we are led to the constraint:

$$V_B \leq \frac{C(1-\tau)}{r}. \tag{9}$$

As for the second property in (8), we observe that if V_B was chosen by the firm, then the total value of the firm v would be maximized by setting V_B as low as possible. Nevertheless, because equity has limited liability, then V_B cannot be arbitrary small, but $E(V,C,T)$ must be nonnegative. Note that $E(V,C,\infty) = V - \frac{C(1-\tau)}{r} \geq 0$ under the following constraint

$$V \geq \frac{C(1-\tau)}{r}. \tag{10}$$

A natural constraint on V_B is $V_B < V$, indeed, if not, the optimal stopping time would necessarily be $T = 0$ and then

$$E(V,C,T) = V - \frac{(1-\tau)C}{r} + \left[\frac{(1-\tau)C}{r} - V_B\right] = V - V_B < 0.$$

Finally $E(V,C,T) \geq 0$ for all $V \geq V_B$.

Proposition 2.3 *The endogenous failure level is*

$$V_B(C;\delta,\tau) = \frac{C(1-\tau)}{r}\frac{\lambda}{\lambda+1}, \tag{11}$$

where λ is given by (6).

Proof: In order to obtain the endogenous failure level V_B we maximize the function (7), which turns in maximizing

$$V_B \mapsto \left(\frac{(1-\tau)C}{r} - V_B\right)\left(\frac{V_B}{V}\right)^{\lambda}.$$

This is a concave function achieving its maximum when $(1-\tau)C\lambda = (\lambda+1)rV_B$. •

Remark 2.4 *Note that (11) satisfies the smooth pasting condition (see [7] footnote 60):*

$$\frac{\partial E}{\partial V}|_{V=V_B} = 0.$$

We observe that the equity function is convex w.r.t. firm's current assets value V, as constraint (9) is satisfied:

$$V_B(C;\delta,\tau) < \frac{(1-\tau)C}{r}.$$

Further (11) has to satisfy $V_B \leq V$, therefore the following inequality holds:

$$\frac{C(1-\tau)}{r}\frac{\lambda}{\lambda+1} \leq V. \tag{12}$$

Remark 2.5 *As a particular case when $\delta = 0$ we obtain Leland [5], where $\lambda = \frac{2r}{\sigma^2}$*

$$E(V, C, V_B) = V - \frac{(1-\tau)C}{r} + \left(\frac{(1-\tau)C}{r} - V_B\right)\left(\frac{V_B}{V}\right)^{2r/\sigma^2},$$

and the failure level is

$$V_B(C; 0, \tau) = \frac{C(1-\tau)}{r + \frac{1}{2}\sigma^2}. \tag{13}$$

Since the application $\delta \mapsto \frac{\lambda}{\lambda+1}$ is decreasing, (13) is greater than (11) for any value of τ:

$$V_B(C; \delta, \tau) = \frac{C(1-\tau)}{r}\frac{\lambda}{\lambda+1} < V_B(C; 0, \tau) = \frac{C(1-\tau)}{r + \frac{1}{2}\sigma^2}, \tag{14}$$

The failure level $V_B(C; \delta, \tau)$ is decreasing with respect to τ, r, σ^2 and proportional to the coupon C, for any value of δ. We note that the dependence of $V_B(C; \delta, \tau)$ on all parameters τ, r, σ^2, C is affected by the choice of the parameter δ. In fact the application $\delta \mapsto \frac{\partial V_B(C;\delta,\tau)}{\partial \tau}$ is negative and increasing, while $\delta \mapsto \frac{\partial V_B(C;\delta,\tau)}{\partial C}$ is positive and decreasing: thus introducing a dividend $\delta > 0$ implies a lower reduction (increase) of the optimal failure level as a consequence of a higher tax rate (coupon), if compared to the case $\delta = 0$. Similarly a change in the risk free rate r or in the riskiness σ^2 of the firm has a different impact on $V_B(C; \delta, \tau)$ depending on the choice of δ.

In line with the results in [5] the failure level $V_B(C; \delta, \tau)$ in (11) is independent of both firm's assets value V and α, the fraction of firm value which is lost in the event of bankruptcy (since the strict priority rule holds). The choice of the optimal failure level $V_B(C; \delta, \tau)$ is a consequence of equity holders maximizing behaviour: this is why it is independent of α. In order to find the optimal failure level equity holders face the problem of maximizing $V_B \mapsto E(V, C, V_B)$ given by (7) and equity value is not affected by bankruptcy costs since the strict priority rule holds. Equity holders will get nothing at bankruptcy, and this value is not affected by the parameter α: only debt holders will bear bankruptcy costs.

2.2 Expected Time to Default

We have proved that introducing dividends has an actual influence on the endogenous failure level $V_B(C; \delta, \tau)$: in particular we showed that a positive dividend δ lowers the failure boundary when we consider the coupon being fixed. Also when the coupon is chosen to maximize the total value of the firm, the optimal failure level $V_B^*(V; \delta, \tau)$ reduces as a consequence of a positive dividend, as we will show in subsection 4. Consistent with our base case parameters' values, Table 2 gives an idea of the magnitude of this reduction in terms of new optimal default triggering level. An interesting point should be to analyze not only the influence of dividends on the failure level, but also on expected time to default. How long does it take for V to reach the failure level V_B? Do dividends have a quantitatively significant effect on expected time to default or not? Since we are considering a framework with infinite horizon, it should be interesting to analyze whether introducing dividends will have a significant influence on the postponement of default. We know that firm's activities value evolves as a log-normal variable, thus the expected time for process V_t to reach the failure level can be studied as shown in the following proposition.

Proposition 2.6 *Let T_b defined in Proposition 2.1. Consider $\mu := r - \delta - \frac{1}{2}\sigma^2$ and $b := \log \frac{V_B}{V}$, with $V \geq V_B$. The following holds:*

- *if $\mu > 0$, $\mathbb{E}_V[T_b] = -\left(\frac{V_B}{V}\right)^{\frac{2\mu}{\sigma^2}} \frac{\log \frac{V_B}{V}}{\mu}$,*
- *if $\mu < 0$, $\mathbb{E}_V[T_b] = \frac{\log \frac{V_B}{V}}{\mu}$.*

Proof: The result follows by Proposition 2.1 and

$$\mathbb{E}_V[T_b] = -\frac{\partial \mathbb{E}[e^{-\gamma T_b}]}{\partial \gamma}|_{\gamma=0}.$$

•

When the drift μ is positive, the expected time to default is decreasing w.r.t. μ, thus increasing w.r.t. dividend δ. In the opposite case, meaning $\mu < 0$, $\mathbb{E}_V[T_b]$ is instead increasing w.r.t. the drift term μ. Dividends will postpone bankruptcy through rising the expected time to default: firm's activities value will take a longer time to reach the failure level. Equity value is increasing w.r.t. δ: as a consequence, the inability of the firm to cover its debt obligations (meaning equity equal to 0) will be later.

3 Comparative statics of financial variables

In this section we aim at analyzing the dependence of all financial variables on C, δ, τ at the endogenous failure level $V_B(C;\delta,\tau)$. By substituting its expression (11) into equity, debt and total value of the firm, we obtain the following functions:

$$E : (C;\delta,\tau) \mapsto V - \frac{(1-\tau)C}{r} + \frac{(1-\tau)C}{r}\frac{1}{\lambda+1}\left(\frac{C(1-\tau)}{rV}\frac{\lambda}{\lambda+1}\right)^{\lambda} \tag{15}$$

$$D : (C;\delta,\tau) \mapsto \frac{C}{r} - \frac{C}{r}\left(1-(1-\alpha)(1-\tau)\frac{\lambda}{\lambda+1}\right)\left(\frac{C(1-\tau)}{rV}\frac{\lambda}{\lambda+1}\right)^{\lambda} \tag{16}$$

$$v : (C;\delta,\tau) \mapsto V + \frac{\tau C}{r} - \frac{C}{r}\left(\tau+\alpha(1-\tau)\frac{\lambda}{\lambda+1}\right)\left(\frac{C(1-\tau)}{rV}\frac{\lambda}{\lambda+1}\right)^{\lambda}. \tag{17}$$

Figure 1 shows the behaviour of equity, debt and total value of the firm given by (15)-(17) as function of dividend δ.

3.1 Equity

We analyze **equity**'s behaviour with respect to δ.

Proposition 3.1 *The equity function (15) is decreasing and convex as function of λ.*

Table 1: **Comparative statics of financial variables.** The table shows the behaviour of all financial variables at $V_B(C;\delta,\tau)$ under constraint (10).

	Limit as		Behaviour w.r.t.		
Variable	$V \to \infty$	$V \to V_B$	C	δ	τ
E	$\sim V - \frac{(1-\tau)C}{r}$	0	Convex, $\searrow$	$\nearrow$	Convex, $\nearrow$
D	$\frac{C}{r}$	$\frac{\lambda C(1-\tau)(1-\alpha)}{r(1+\lambda)}$	Concave, $\cap$-Shaped	$\searrow$	$\nearrow$
v	$\sim V + \frac{\tau C}{r}$	$\frac{\lambda C(1-\tau)(1-\alpha)}{r(1+\lambda)}$	Concave	$\searrow$ [a]	$\nearrow$
R	r	$\frac{r(1+\lambda)}{\lambda(1-\alpha)(1-\tau)}$	Concave	$\nearrow$	$\searrow$
$R-r$	0	$\frac{r(1+\lambda(\alpha+\tau-\alpha\tau))}{\lambda(1-\alpha)(1-\tau)}$	Concave	$\nearrow$	$\searrow$

[a] See Proposition 3.7.

Proof: Equity's behaviour w.r.t. λ is summarized by

$$f(\lambda) = \frac{1}{\lambda+1}\left(\frac{C(1-\tau)}{rV}\frac{\lambda}{1+\lambda}\right)^{\lambda}. \tag{18}$$

The logarithmic derivative of (18) is $\log(\frac{C(1-\tau)}{rV}\frac{\lambda}{\lambda+1})$ which is negative by (12). Moreover

$$f''(\lambda) = \frac{1}{\lambda(1+\lambda)}f(\lambda) + \left(\log\frac{C(1-\tau)}{rV} + \log\frac{\lambda}{1+\lambda}\right)f'(\lambda) > 0, \tag{19}$$

thus equity is decreasing and convex w.r.t. λ. •

As a consequence of Proposition 3.1 and Remark 2.2, equity is increasing w.r.t. δ. Concerning equity's convexity w.r.t. δ, the following result holds.

Proposition 3.2 *The equity function (15) is convex w.r.t.* δ *if*

$$V > V_B(C;\delta,\tau)e^{\frac{2r}{\lambda^2\sqrt{\mu^2+2r\sigma^2}}} \tag{20}$$

where $V_B(C;\delta,\tau)$ *is given by (11).*

Proof: In order to study equity's convexity w.r.t. δ, we evaluate $\partial^2_\delta E$ using (18) and (19), obtaining:

$$\begin{aligned}\partial^2_\delta E &= \left[\frac{1}{\lambda(1+\lambda)}f(\lambda) + \left(\log^2\frac{C(1-\tau)}{rV}\frac{\lambda}{1+\lambda}\right)f(\lambda)\right]\frac{\lambda^2}{\mu^2+2r\sigma^2} \\ &+ f(\lambda)\left(\log\frac{C(1-\tau)}{rV}\frac{\lambda}{1+\lambda}\right)\frac{2r}{(\mu^2+2r\sigma^2)\sqrt{\mu^2+2r\sigma^2}}\end{aligned} \tag{21}$$

Substituting $V_B(C;\delta,\tau) = C(1-\tau)\lambda/(r(1+\lambda))$ and re-arranging terms gives:

$$\partial^2_\delta E = \frac{f(\lambda)}{\mu^2+2r\sigma^2}\left[\frac{\lambda}{1+\lambda} + \log\frac{V_B}{V}\left(\lambda^2\log\frac{V_B}{V} + \frac{2r}{\sqrt{\mu^2+2r\sigma^2}}\right)\right] \tag{22}$$

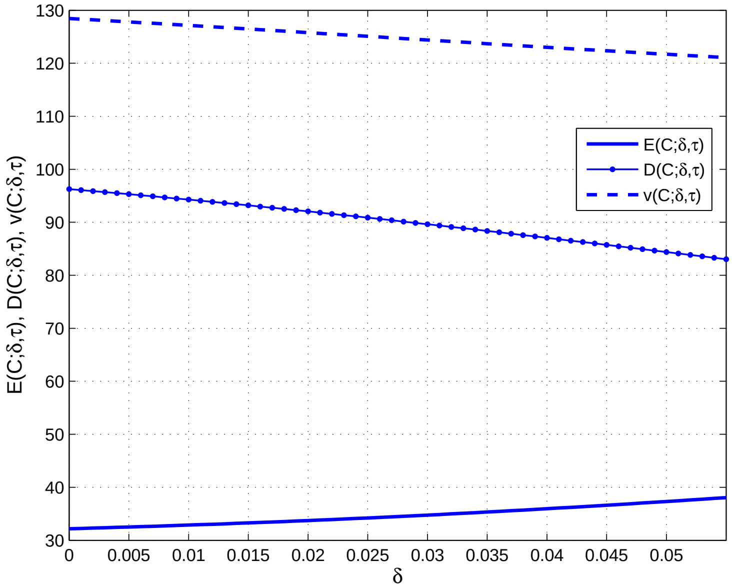

Figure 1: **Equity, debt and total value of the firm as function of δ.** This plot shows the behaviour of equity (15), debt (16) and total value of the firm (17) as function of δ, for a fixed level of coupon $C = 6.5$ satisfying (10). We assume $V = 100$, $r = 6\%$, $\sigma = 0.2$, $\tau = 0.35$, $\alpha = 0.5$.

As $V \geq V_B$ then $\log \frac{V_B}{V} < 0$ for each parameters' choice, then a sufficient condition for equity's convexity w.r.t. δ is

$$\lambda^2 \log \frac{V_B}{V} + \frac{2r}{\sqrt{\mu^2 + 2r\sigma^2}} < 0,$$

which is equivalent to (20). •

Remark 3.3 *Equity's dependence on dividend δ is strictly related to equity's dependence on μ. Observe that $\partial^2_\delta E = \partial^2_\mu E$, since $\partial_\mu \lambda = -\partial_\delta \lambda$ and $\partial^2_\mu \lambda = \partial^2_\delta \lambda$.*
Introducing dividends has a positive effect on equity: this positive effect is even magnified if the initial distance to default is big enough, meaning constraint (20) being satisfied.

Studying equity's behaviour w.r.t. C and τ, we can observe that E in (15) is a function of the product $C(1-\tau)$; thus $E(C;\delta,\tau)$ is: *i)* decreasing and convex w.r.t. coupon C, *ii)* increasing and convex w.r.t. the corporate tax rate τ.

We observe that also in the presence of a dividend $\delta > 0$, equity holders have incentives to increase the riskiness of the firm, since λ decreases with higher volatility. These incentives are higher as dividend increases: Figure 2 shows the behaviour of $\frac{\partial E}{\partial \sigma}$ as function of V, for three different levels of δ. For each value of V, $\frac{\partial E}{\partial \sigma}$ increases with δ, meaning that a higher dividend produces greater incentives for shareholders to increase the riskiness of the firm, thus rising potential agency costs due to incentive compatibility problem between shareholders and debt holders.

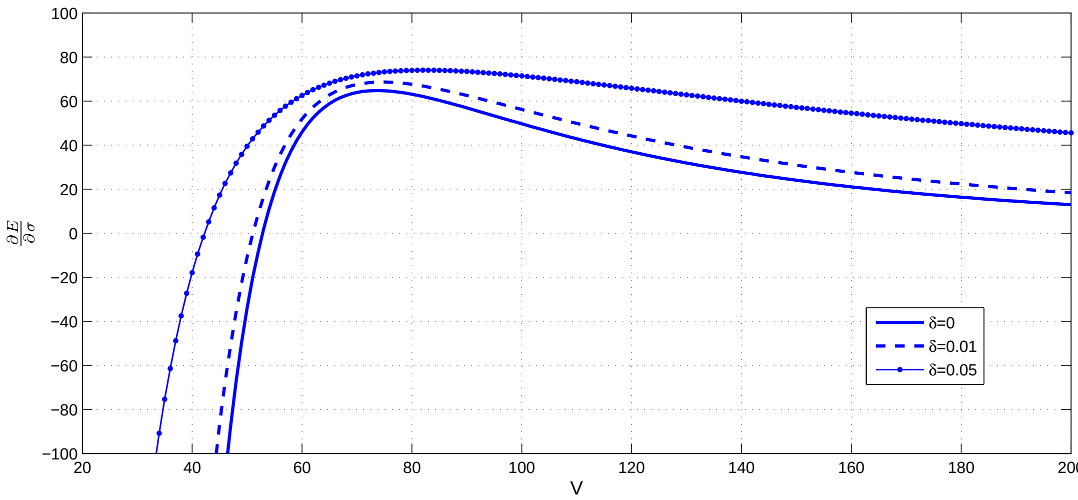

Figure 2: **Effect of a change in σ on equity value.** This plot shows the behaviour of $\frac{\partial E}{\partial \sigma}$ as function of firm's current assets value V, for a fixed level of coupon $C = 6.5$ and different values of δ. We consider $r = 0.06, \sigma = 0.2, \alpha = 0.5, \tau = 0.35$. Equity value is given by (15).

3.2 Debt and Yield Spread

We consider now the **debt function** $D(C; \delta, \tau)$ in (16). The application $D(C; \delta, \tau)$ is concave w.r.t. coupon C, allowing to analyze the maximum capacity of debt of the firm as it is shown in the following Proposition.

Proposition 3.4 *The application $C \mapsto D(C; \delta, \tau)$ is concave and it achieves a maximum at*

$$C_{max}(V, \delta, \tau) = \frac{rV(1+\lambda)}{\lambda(1-\tau)} \left(\frac{1}{\lambda(\tau + \alpha(1-\tau)) + 1} \right)^{\frac{1}{\lambda}}. \tag{23}$$

$C_{max}(V, \delta, \tau)$ represents the maximum capacity of the firm's debt. Substituting this value for the coupon into debt function $D(C; \delta, \tau)$ and simplifying yields:

$$D_{max}(V, \delta, \tau) = \frac{V}{1-\tau} \left(\frac{1}{\lambda(\tau + \alpha(1-\tau)) + 1} \right)^{\frac{1}{\lambda}}. \tag{24}$$

Equation (24) represents the debt capacity of the firm: the maximum value that debt can achieve by choosing the coupon C. Not surprisingly the debt capacity of the firm is proportional to firm's current assets value V, decreases with higher bankruptcy costs α and increases if the corporate tax rate rises. In the presence of a positive dividend, if τ changes, its effect on debt capacity is lower than in case $\delta = 0$, since $\delta \mapsto \frac{\partial D_{max}(V;\delta,\tau)}{\partial \tau}$ is decreasing.

Under constraint (10), as δ increases, debt decreases as shown in the following Proposition.

Proposition 3.5 *Debt value $D(C; \delta, \tau)$ defined in (16) is a decreasing function of dividend δ.*

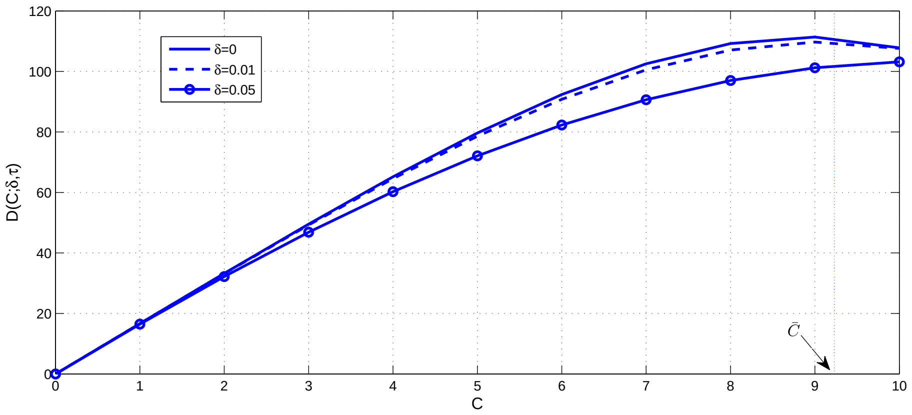

Figure 3: **Debt value as function of the coupon.** This plot shows the behaviour of debt value given in (16) as function of coupon payments C, for different levels of δ. We assume $V = 100$, $r = 0.06$, $\sigma = 0.2$, $\tau = 0.35$, $\alpha = 0.5$. We consider three different levels of δ: $\delta = 0$, $\delta = 0.01$, $\delta = 0.05$. The value $\bar{C} = \frac{rV}{(1-\tau)}$ is the maximum value that coupon C can assume due to constraint $C(1-\tau) - rV < 0$.

Proof: It is enough to study the monotonicity of debt function with respect to λ. Debt's dependence on λ is the opposite of

$$g(\lambda) = \left(1 - (1-\alpha)(1-\tau)\frac{\lambda}{\lambda+1}\right)\left(\frac{C(1-\tau)}{rV}\frac{\lambda}{\lambda+1}\right)^{\lambda},$$

its log-derivative being

$$h(\lambda) = \frac{g'}{g}(\lambda) = \left(\log\frac{\lambda}{\lambda+1} + \frac{\alpha+\tau-\alpha\tau}{1+\lambda(\alpha+\tau-\alpha\tau)} + \log\frac{C(1-\tau)}{rV}\right).$$

Function h admits a positive derivative

$$h'(\lambda) = \frac{1}{\lambda(\lambda+1)} - \left(\frac{\alpha+\tau-\alpha\tau}{1+\lambda(\alpha+\tau-\alpha\tau)}\right)^2 = \frac{\lambda(\alpha+\tau-\alpha\tau)(2-\alpha-\tau+\alpha\tau)+1}{\lambda(\lambda+1)(1+\lambda(\alpha+\tau-\alpha\tau))^2} \geq 0.$$

This inequality implies that $h(\lambda)$ increases from $-\infty$ to $\log\frac{C(1-\tau)}{rV}$, which is negative by (10). •

As dividend δ increases, the maximum capacity of debt reduces (the application $\lambda \mapsto D_{max}(V;\delta,\tau)$ is increasing) and C_{max} increases. Equity holders have incentives to increase the riskiness of the firm since $\frac{\partial E}{\partial\sigma} > 0$, while the opposite happens for debt holders, $\frac{\partial D}{\partial\sigma} < 0$: higher volatility decreases debt value. The "asset substitution" problem still exists. There is an incentive compatibility problem between debt holders and equity holders: once debt is issued, shareholders will benefit from an increase in σ, through transferring value from debt to equity (also if equity is not exactly an ordinary call option, see [6] footnote 29). In Figure 4 we analyze which is the effect of a change in the volatility level σ on equity and debt value, when dividends are introduced. Following [6] we study the magnitude of this effect as function of V. Considering three different levels of dividend $\delta = 0, 0.01, 0.05$ we compute $\frac{\partial E}{\partial\sigma}$, $\frac{\partial D}{\partial\sigma}$ and analyze them as functions of V. The firm will bear agency costs for the range of values V such that $\frac{\partial E}{\partial\sigma} > 0$,

$\frac{\partial D}{\partial \sigma} < 0$. Figure 4 shows that introducing dividends increases this range, thus rising the problem of adverse incentives between debt holders and equity holders and so potential agency costs for the firm. Considering a positive δ, as V becomes higher, the incentive compatibility problem becomes more difficult to solve, since the distance between $\frac{\partial E}{\partial \sigma}$ and $\frac{\partial D}{\partial \sigma}$ increases with a greater δ. This is still true also considering a "pure" Modigliani-Miller framework with zero tax benefits and bankruptcy costs, where $\frac{\partial E}{\partial \sigma} = -\frac{\partial D}{\partial \sigma}$. In such a case the conflict approaches a zero sum game as found in [6] and the introduction of dividends increases its magnitude (see Figure 5).

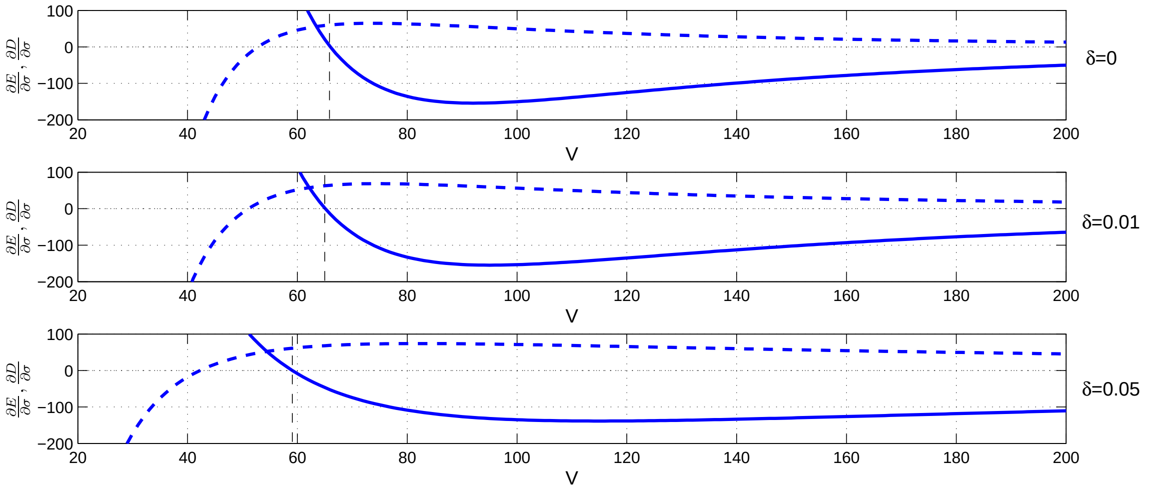

Figure 4: **Effect of a change in σ on equity and debt values.** This plot shows the behaviour of $\frac{\partial E}{\partial \sigma}$ (dashed line) and $\frac{\partial D}{\partial \sigma}$ (solid line) as function of firm's current assets value V, for a fixed level of coupon $C = 6.5$. We consider $r = 0.06, \sigma = 0.2, \alpha = 0.5, \tau = 0.35$. Equity value is given by (15), debt value by (16).

Finally, a higher coupon C has a positive effect on the interest rate paid by risky debt, **yield**, defined as

$$R(C; \delta, \tau) := \frac{C}{D(C; \delta, \tau)}, \tag{25}$$

with $D(C; \delta, \tau)$ given in (16).

Actually yield $R(C; \delta, \tau)$ is increasing as function of C and decreasing as function of τ. A higher corporate tax rate τ will reduce both yield $R(C; \delta, \tau)$ and **yield spread** $R(C; \delta, \tau) - r$ by rising debt (lowering the endogenous failure level $V_B(C; \delta, \tau)$, see also [5] footnote 22): this follows by the relation $\partial_\tau R(C; \delta, \tau) = -C \frac{\partial_\tau D(C;\delta,\tau)}{(D(C;\delta,\tau))^2}$.

Proposition 3.6 *The function yield $R(C; \delta, \tau)$ defined in (25) is increasing w.r.t. dividend δ.*

Proof: As D is an increasing function of λ and $\partial_\lambda R = -C \frac{\partial_\lambda D}{D^2}$, we obtain that R is a decreasing function of λ. Thus by Remark 2.2 R is increasing w.r.t. δ. •

Observe that R can be expressed as:

$$R : (C; \delta, \tau) \mapsto r \bar{R} \left(\frac{C}{V} \right), \tag{26}$$

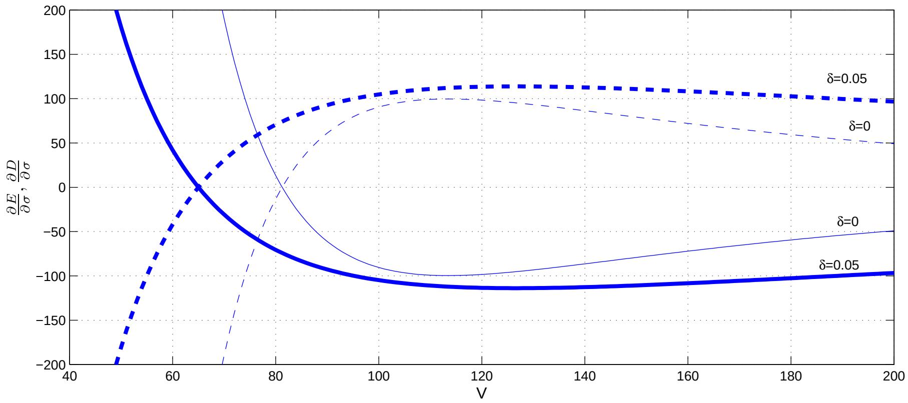

Figure 5: **Effect of a change in σ on equity and debt values when $\alpha = \tau = 0$.** This plot shows the behaviour of $\frac{\partial E}{\partial \sigma}$ (dashed line) and $\frac{\partial D}{\partial \sigma}$ (solid line) as function of firm's current assets value V, for a fixed level of coupon $C = 6.5$. We consider $r = 0.06, \sigma = 0.2, \alpha = 0.5, \tau = 0.35$ and two different levels of dividend $\delta = 0, 0.05$. Equity value is given by (15), debt value by (16).

with

$$\bar{R}\left(\frac{C}{V}\right) = \left[1 - \left(\frac{C}{V}\right)^{\lambda} \left(\frac{(1-\tau)}{r}\frac{\lambda}{\lambda+1}\right)^{\lambda} \left(1 - (1-\alpha)(1-\tau)\frac{\lambda}{\lambda+1}\right)\right]^{-1}. \qquad (27)$$

As increasing function of the ratio $\frac{C}{V}$, the term $\bar{R}\left(\frac{C}{V}\right)$ represents the *risk-adjustment factor* paid to debt holders. Introducing dividends rises debt's volatility as Figure 6 shows: as a consequence, the compensation paid by the firm to debt holders for the risk assumed must be higher, and this is why $\bar{R}\left(\frac{C}{V}\right)$ increases.

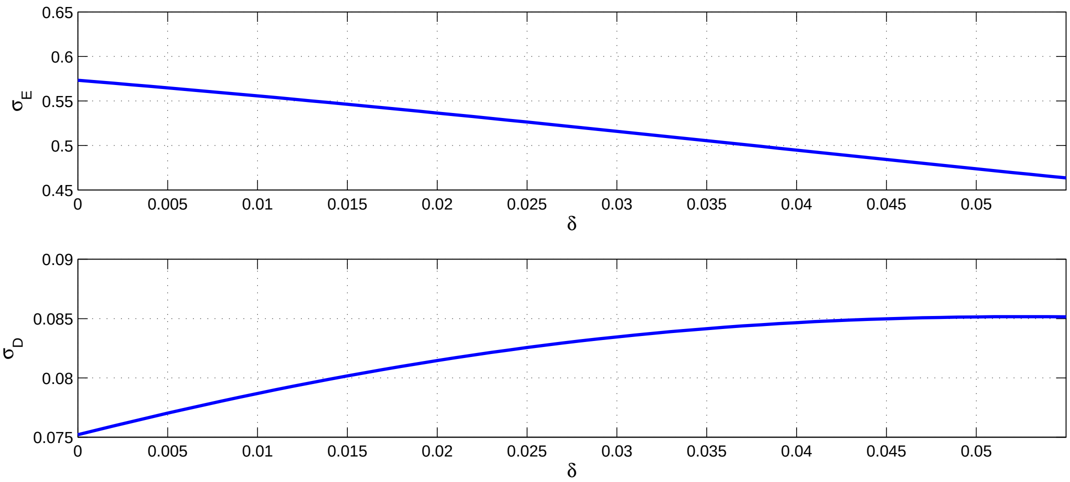

Figure 6: **Volatility of Equity and Debt.** This plot shows the behaviour of equity and debt volatility σ_E, σ_D as function of dividends, for a fixed level of coupon $C = 6.5$. We consider $V = 100, r = 0.06, \sigma = 0.2, \alpha = 0.5, \tau = 0.35$. By Ito calculus formula we derive the behaviour of equity and debt volatility σ_E, σ_D.

When $V \to \infty$, yield spread $R - r$ approaches to r, $\frac{\partial D}{\partial \sigma} \to 0$ and debt becomes risk free: this is exactly as in [5], since in such a case, the hypothesis of debt being redeemed in full becomes quite certain and this is not affected by the choice of dividend δ. A positive dividend will instead rise R in case $V \to V_B$, considering a "pure" Modigliani-Miller [9] framework: if $\alpha = \tau = 0$, as V approaches the failure level V_B, $R \to r(1 + \frac{1}{\lambda})$, while in case $\delta = 0$ we have $R \to r + \frac{1}{2}\sigma^2$. If there are no bankruptcy costs or tax benefits of debt, introducing dividends allows to have yield exceeding the risk free rate r by more than $\frac{1}{2}\sigma^2$, since $\frac{r}{\lambda} > \frac{1}{2}\sigma^2$, thus providing to bondholders a higher compensation for risk.

3.3 Total Value

The **total value of the firm** $v(C; \delta, \tau)$ in (17) is a concave function of coupon C and an increasing function of corporate tax rate τ. The following proposition shows the behaviour of the total value of the firm with respect to dividend δ.

Proposition 3.7 *The total value $v(C; \delta, \tau)$ defined in (17) is decreasing w.r.t. δ if*

$$V > V_B(C; \delta, \tau) e^{\frac{x+\alpha}{x+\lambda(x+\alpha)}} \tag{28}$$

with $x = \frac{\tau}{1-\tau}$, and $V_B(C; \delta, \tau)$ given in (11).

Proof: The behaviour of $v(C; \delta, \tau)$ is the one of the following:

$$G : \lambda \mapsto (\tau + \lambda(\tau + \alpha(1 - \tau))) \frac{1}{\lambda + 1} \left(\frac{C(1 - \tau)}{rV} \frac{\lambda}{\lambda + 1} \right)^{\lambda},$$

which satisfies, letting $x = \frac{\tau}{1-\tau}$:

$$\frac{G'}{G}(\lambda) = h(\lambda) = \frac{x + \alpha}{x + \lambda(x + \alpha)} + \log \left(\frac{C(1 - \tau)}{rV} \frac{\lambda}{\lambda + 1} \right) = \frac{x + \alpha}{x + \lambda(x + \alpha)} + \log \frac{V_B(C; \delta, \tau)}{V}.$$

Actually, the behaviour of G is given by the sign of $h(\lambda)$, with $\frac{x+\alpha}{x+\lambda(x+\alpha)} > 0$ and $\log \frac{V_B(C;\delta,\tau)}{V} < 0$.

Thus, in case

$$V > V_B(C; \delta, \tau) e^{\frac{x+\alpha}{x+\lambda(x+\alpha)}}$$

we have $h(\lambda) < 0$ and the total value of the firm is decreasing w.r.t. δ. •

The economic intuition behind constraint (28) being satisfied, is that if the initial value of the firm V is sufficiently greater than the failure level, $\delta \mapsto v(C; \delta, \tau)$ is decreasing since introducing dividends makes bankruptcy more likely.

Figure 7 shows the cumulative probability $F(s)$ of going bankruptcy in the interval $[0, s)$:

$$F(s) = N \left(\frac{b - \mu s}{\sigma \sqrt{s}} \right) + e^{2b\mu/\sigma^2} N \left(\frac{b + \mu s}{\sigma \sqrt{s}} \right), \tag{29}$$

where $N(\cdot)$ is the normal cumulative probability function and $b := \log \frac{V_B}{V}$, see also Equation (15) in [6]. As observed in [6], while the probability of going bankruptcy is negligible if we consider only the first few years, it rises if we look at a longer period of time. In line with [6] footnote 27, Figure 7 shows that the probability of default is quite dependent on the drift $r - \delta$ assumed for the process V_t. What we want to study is dividend influence on the cumulative probability of going bankruptcy $F(s)$.

As an example we consider three different levels $\delta = 0, 0.01, 0.05$. Looking at cumulative probability over a period longer than 10 years shows that introducing dividends makes debt riskier, since it strongly increases the likelihood of default. Considering a period of 20 years, the probability of default is 41% with a dividend $\delta = 0.05$ rather than 21% in case $\delta = 0$.

At the same time, we observe an opposite behaviour of $F(s)$ if we look at very short time intervals (approximately less than 6 years with our base case values): the cumulative probability of going bankruptcy is less with dividends if compared to the case $\delta = 0$. Intuitively we could think of this in the following way: when the interval $[0, s)$ is very short, the probability of going bankruptcy is low, thus default is not imminent and this is independent of the dividend level. In such a case, introducing dividends will lower $F(s)$ since its only effect is to make the failure level farer.

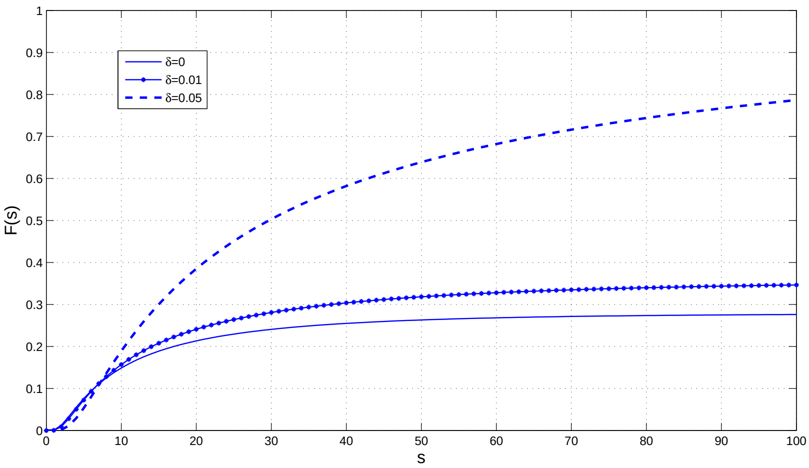

Figure 7: **Cumulative Probability of Default.** This plot shows the cumulative probability of default $F(s)$ over the period $(0, s]$, considering Equation (29). The plot shows $F(s)$ for three different values of parameter δ. We consider: $\delta = 0, 0.01, 0.05$. Base case parameters' values are $V = 100, r = 0.06, \sigma = 0.2, \alpha = 0.5, \tau = 0.35$; the coupon is chosen optimally.

4 Optimal Leverage

Now we turn to the optimization of the total value of the firm $v(C; \delta, \tau)$ with respect to the coupon C, depending on the failure level $V_B(C; \delta, \tau)$ in (11). This application is concave since $A := \frac{\tau}{r} + \alpha \frac{\lambda(1-\tau)}{r(\lambda+1)} > 0$ and $\lambda > 0$, therefore the following result holds.

Proposition 4.1 *For any fixed δ, τ, the optimal coupon is:*

$$C^*(V;\delta,\tau) = \frac{rV(\lambda+1)}{\lambda(1-\tau)} \left(\frac{\tau}{\lambda(\tau+\alpha(1-\tau))+\tau} \right)^{\frac{1}{\lambda}}. \tag{30}$$

We observe that $C^*(V;\delta,\tau) < C_{max}(V;\delta,\tau)$, where C_{max} is defined in (23). Moreover, this max-coupon satisfies $V > \frac{(1-\tau)C_{max}}{r}\frac{\lambda}{\lambda+1}$.
The optimal coupon $C^*(V;\delta,\tau)$ is an increasing function of τ. In fact

$$\frac{\partial C^*(V,\delta,\tau)}{\partial \tau} = \left(\frac{1}{1-\tau} + \frac{\alpha}{\tau(\tau(1+\lambda)+\alpha\lambda(1-\tau))} \right) C^*(V,\delta,\tau) > 0.$$

Replacing (30) in (11) yields the optimal failure level

$$V_B^*(V;\delta,\tau) = V\left(\frac{\tau}{\lambda(\tau+\alpha(1-\tau))+\tau} \right)^{\frac{1}{\lambda}}. \tag{31}$$

Remark 4.2 *In case $\delta = 0$, we have $\lambda = \frac{2r}{\sigma^2}$ and we get the same results as in [5]:*

$$V_B^*(V;0,\tau) = V\left(\frac{\tau\sigma^2}{2r(\tau+\alpha(1-\tau))+\tau\sigma^2} \right)^{\frac{\sigma^2}{2r}}. \tag{32}$$

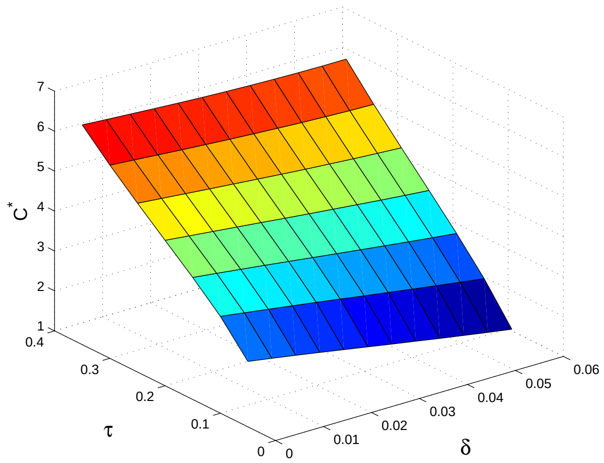

Figure 8: **Optimal Coupon.** This plot shows the behaviour of optimal coupon $C^*(V;\delta,\tau)$ as function of dividend δ and corporate tax rate τ. We consider $V = 100$, $r = 0.06, \sigma = 0.2, \alpha = 0.5$.

Proposition 4.3 *Consider the optimal failure level (31). The following results hold:*
i) $\delta \mapsto V_B^*(V;\delta,\tau)$ is a decreasing function;
ii) $\tau \mapsto V_B^*(V;\delta,\tau)$ is an increasing function.

Proof: i) Using Remark 2.2, it is enough to study the following function

$$F : \lambda \mapsto -\frac{1}{\lambda} \log \left(\frac{\tau + \lambda(\tau + \alpha(1-\tau))}{\tau} \right).$$

Taking the derivative w.r.t. λ, we obtain:

$$F'(\lambda) = \frac{1}{\lambda^2} \left(\log(1+z) - \frac{z}{1+z} \right),$$

with $z := \lambda \left(1 + \alpha \left(\frac{1-\tau}{\tau}\right)\right)$. It is sufficient to study the sign of

$$G : z \mapsto \log(z+1) - \frac{z}{1+z},$$

with $z \in \left[0, \frac{2r}{\sigma^2}\left(1 + \alpha\left(\frac{1-\tau}{\tau}\right)\right)\right]$. Since $G(0) = 0$ and $G'(z) \geq 0$, for any z the function F is increasing. Finally $\delta \mapsto V_B^*(V;\delta,\tau)$ is decreasing.

ii) The result follows by:

$$\frac{\partial V_B^*(V;\delta,\tau)}{\partial \tau} := \frac{\partial V_B^*(C^*(V;\delta,\tau))}{\partial \tau} = \frac{\partial V_B^*(C^*(V;\delta,\tau))}{\partial C^*} \frac{\partial C^*(V;\delta,\tau)}{\partial \tau} > 0.$$

•

Introducing a positive dividend into (31) has an actual influence: $\delta \mapsto V_B^*(V;\delta,\tau)$ is a decreasing function for any value of τ, while $\tau \mapsto V_B^*(V;\delta,\tau)$ is increasing for any value of δ. Similarly optimal coupon $C^*(V;\delta,\tau)$ given by (30) will benefit from a higher corporate tax rate and decrease w.r.t. dividend δ, as Figure 8 shows.

Table 2: **Effect of dividend δ on all financial variables at the optimal leverage ratio.** Base case parameters' values: $V = 100$, $\sigma = 0.2$, $\tau = 0.35$, $r = 0.06$, $\alpha = 0.5$. The first and last rows of the table show Leland's framework with his base case parameters' values, in particular with $\delta = 0$ and 0.055 (in this last case the drift $r - \delta$ of process $V_.$ is exactly as in [6]). R^*, L^* are in percentage (%), $R^* - r$ is in basis points (bps).

δ	C^*	D^*	R^*	$R^* - r$	E^*	V_B^*	v^*	L^*
0	6.5010	96.2742	6.7526	75.26	32.1675	52.8204	128.4417	74.9556
0.005	6.4592	94.9261	6.8045	80.45	32.8775	51.6352	127.8036	74.2750
0.01	6.4188	93.5455	6.8617	86.17	33.6038	50.4201	127.1493	73.5714
0.015	6.3803	92.1378	6.9248	92.48	34.3437	49.1799	126.4815	72.8469
0.02	6.3444	90.7095	6.9942	99.42	35.0940	47.9207	125.8034	72.1041
0.025	6.3116	89.2682	7.0703	107.03	35.8507	46.6494	125.1189	71.3467
0.03	6.2826	87.8228	7.1537	115.37	36.6095	45.3743	124.4323	70.5788
0.035	6.2581	86.3826	7.2446	124.46	37.3657	44.1039	123.7482	69.8051
0.04	6.2386	84.9573	7.3433	134.33	38.1143	42.8473	123.0716	69.0308
0.045	6.2248	83.5563	7.4498	144.98	38.8508	41.6132	122.4071	68.2610
0.05	6.2169	82.1886	7.5642	156.42	39.5705	40.4099	121.7592	67.5010
0.055	6.2154	80.8621	7.6864	168.64	40.2696	39.2446	121.1317	66.7555

The capital structure of the firm is strongly affected by dividends: Tables 2 and 3 show the behaviour of all financial variables at optimal leverage ratio, when the parameter δ moves away from zero. Consistent with our base case, these tables report both numerical results and a qualitative analysis.

Let $y := \lambda(\tau + \alpha(1-\tau))$, the following holds:

$$
\begin{aligned}
C^* &= \frac{rV(1+\lambda)}{\lambda(1-\tau)}\left(\frac{\tau}{y+\tau}\right)^{\frac{1}{\lambda}} && (33)\\
V_B^* &= V\left(\frac{\tau}{y+\tau}\right)^{\frac{1}{\lambda}} && (34)\\
D^* &= \frac{V}{\lambda(1-\tau)}\left(\frac{\tau}{y+\tau}\right)^{\frac{1}{\lambda}}\left(\lambda+\frac{y(1-\tau)}{y+\tau}\right) && (35)\\
E^* &= V\left(1-\left(\frac{\tau}{y+\tau}\right)^{\frac{1}{\lambda}}\frac{1}{\lambda}\left(1+\lambda+\frac{\tau}{y+\tau}\right)\right) && (36)\\
R^* &= \frac{r(1+\lambda)}{\lambda+\frac{y(1-\tau)}{y+\tau}}. && (37)
\end{aligned}
$$

Columns 6 and 7 of Table 2 show equity and debt values when the coupon C is chosen to maximize the total value of the firm. Optimal equity value increases with a higher dividend, while optimal debt decreases. These two effects have a different magnitude: δ influence on debt is in fact greater than δ influence on equity, as a consequence the optimal total value of the firm, $v^* := D^* + E^*$, reduces. Now consider optimal leverage ratio, defined as $L^* := \frac{D^*}{v^*}$. The last column of Table 2 shows that increasing dividends decreases optimal leverage ratio L^*, and this effect is more pronounced as δ is higher. Considering our base case, optimal leverage can drop from approximately 75% to 66.75% passing from $\delta = 0$ to $\delta = 0.055$. Firms paying dividends can choose very different optimal leverage ratios, depending on the level of their riskiness. Figure 9 shows optimal leverage as function of δ for three different values of σ. For each level of δ, optimal leverage ratio decreases as σ rises. Observe also that L^* is decreasing w.r.t. δ for each level of σ, but this reduction in optimal leverage is lower as the riskiness of the firm rises.

Recall from Section 2 that introducing dividends rises the probability of going bankruptcy given by (29): as a consequence, when δ rises, optimal yield R^* and optimal yield spread $R^* - r$ increase. Leland [5] observes that as bankruptcy costs rise, surprisingly optimal yield spread reduces when the coupon is chosen optimally. This is due to the fact that a higher α will decrease the optimal coupon. Our analysis shows that when a dividend is introduced, optimal yield spreads are decreasing w.r.t. α for each level of δ (see Figure 10). Dividends influence on optimal yield spreads is higher as α reduces. Considering as extreme cases $\delta = 0$ and $\delta = 0.055$, optimal yield spread rises from 71.6778 bps (basis points) to 156.2624 bps with $\alpha = 0.8$, from 75.2554 bps to 168.6365 bps with $\alpha = 0.5$ and from 81.2559 bps to 188.424 bps with $\alpha = 0.2$.

We now turn to the study of tax deduction τ influence on all financial variables. Tables 4 and 5 show the behaviour of all financial variables at optimal coupon level C^* for different values of the corporate tax rate τ when a dividend $\delta = 0.01$ is introduced. As the tax deduction increases, all financial variables, except equity, will benefit from this. This result extends Table II in [5] since it allows for a dividend $\delta > 0$. Concerning the optimal failure level $V_B^*(V; \delta, \tau)$ we observe that by (31) the corporate tax rate τ has no influence on the optimal failure level at optimal

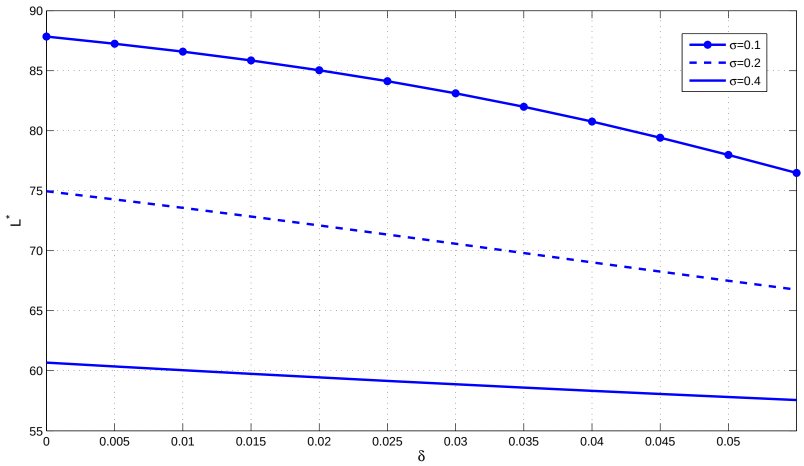

Figure 9: **Optimal leverage ratio as function of δ.** This plot show optimal leverage ratio L^* as function of δ for three different levels of volatility σ. We consider $V = 100$, $r = 0.06$, $\alpha = 0.5$, $\tau = 0.35$.

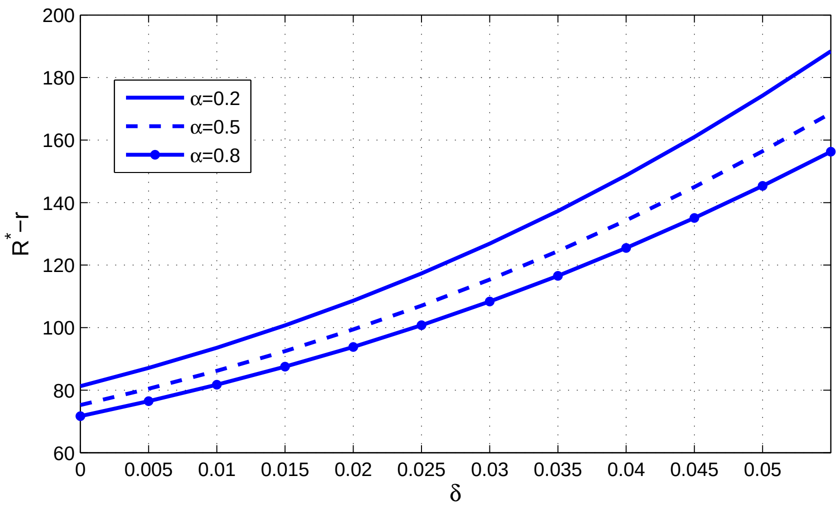

Figure 10: **Optimal spreads as function of δ.** This plot show optimal spreads $R^* - r$ as function of δ for three different levels of α. We consider $V = 100$, $r = 0.06$, $\sigma = 0.2$, $\tau = 0.35$.

Table 3: **Effect of dividend δ on all financial variables at the optimal leverage ratio.** The table shows for each financial variable the effect of increasing δ. Considering our base case, we report the sign of change in each variable as the dividend moves away from 0.

Financial Variables	C^*	D^*	R^*	R^*-r	E^*	V_B^*	v^*	L^*
Sign of change as $\delta \nearrow$	< 0	< 0	> 0	> 0	> 0	< 0	< 0	< 0

leverage ratio in case $\alpha = 0$. The same holds for equity value E^* at optimal leverage ratio: a change in the corporate tax rate has no effect on equity value in the absence of bankruptcy costs.

Table 4: **Effect of a change in the corporate tax rate τ on all financial variables at the optimal leverage ratio.** This table considers a case in which a dividend δ is introduced ($\delta = 0.01$) and studies the effect of a change in the corporate tax τ. R^*, L^* are in percentage (%), R^* is in basis points (bps).

τ	C^*	D^*	R^*	$R^* - r$	E^*	V_B^*	v^*	L^*
0.35	6.4188	93.5455	6.8617	86.17	33.6038	50.4201	127.1493	73.5714
0.30	5.7425	85.0717	6.7502	75.02	35.7472	48.5775	120.8189	70.4126
0.25	5.1154	77.0816	6.6363	63.63	38.3728	46.3633	115.4544	66.7637
0.20	4.5132	69.2273	6.5193	51.93	41.6807	43.6322	110.9081	62.4187
0.15	3.9071	61.0635	6.3984	39.84	46.0189	40.1338	107.0824	57.0248

Table 5: **Effect of corporate tax rate τ on all financial variables at the optimal leverage ratio.** The table shows for each financial variable the effect of increasing τ for fixed $\delta = 0.01$. Considering our base case, we report the sign of change in each variable as the corporate tax rate increases.

Financial Variables	C^*	D^*	R^*	R^*-r	E^*	V_B^*	v^*	L^*
Sign of change as $\tau \nearrow$	> 0	> 0	> 0	> 0	< 0[a]	> 0[a]	> 0	> 0

[a] No effect if $\alpha = 0$.

5 Conclusions

Adding dividends has an actual influence on all financial variables: for an arbitrary coupon level C, a positive dividend increases equity and decreases both debt and total value of the firm, making bankruptcy more likely. Dividends strongly modify the influence of all parameters r, τ, C, σ^2 on the endogenous failure level $V_B(C; \delta, \tau)$ (magnitude of the change). Concerning optimal capital structure Leland's [5] results show too high leverage ratios (and/or too low yield spreads): assuming $\delta > 0$ allows to overcome this, providing lower optimal leverage ratios (and higher yield spreads).

References

[1] Dorobantu, D.: Modélisation du risque de défaut en entreprise. Ph.D. Thesis, University of Toulouse. http://thesesups.ups-tlse.fr/148/1/Dorobantu_Diana.pdf. (2007)

[2] Dorobantu, D., Mancino, M.E., Pontier, M.: Optimal strategies in a risky debt context. Stochastics An International Journal of Probability and Stochastic Processes. **81:3**, 269–277 (2009)

[3] El Karoui, N.: Les Aspects Probabilistes du Contrôle Stochastique. Lecture Notes in Mathematics 876, pp. 73-238. Springer-Verlag, Berlin (1981)

[4] Karatzas, I., Shreve, S.: Brownian Motion and Stochastic Calculus. Springer, Berlin, Heidelberg, New York (1988)

[5] Leland, H.E.: Corporate debt value, bond covenant, and optimal capital structure. The Journal of Finance. **49**, 1213–1252 (1994)

[6] Leland, H.E. and Toft, K.B.: Optimal capital structure, endogenous bankruptcy and the term structure of credit spreads. The Journal of Finance. **51**, 987–1019 (1996)

[7] Merton, R. C.: A rational theory of option pricing. Bell Journal of Economics and Management Science. **4**, 141–183 (1973)

[8] Merton, R. C.: On the pricing of corporate debt: the risk structure of interest rates. The Journal of Finance. **29**, 449–470 (1974)

[9] Modigliani, F., Miller, M.: The cost of capital, corporation finance and the theory of investment. American Economic Review. **48**, 267–297 (1958)